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THESIS

Investigation of Alternative Methods
Including Jackknifing for Estimating
Point Availability of a System

Barbaros Aba

September 1981

Thesis Advisor:

D. P. Gaver

Approved for public release; distribution unlimited

Prepared for:
Naval Postgraduate School
Monterey, California

T202021

NAVAL POSTGRADUATE SCHOOL
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| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|--|-----------------------|--|
| 1. REPORT NUMBER NPS55-81-021 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) Investigation of Alternative Methods Including Jackknifing for Estimating Point Availability of a System | | 5. TYPE OF REPORT & PERIOD COVERED Thesis Report Nov 1980 - Sep 1981 |
| 7. AUTHOR(s) Barbaros Aba | | 6. PERFORMING ORG. REPORT NUMBER |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940 | | 12. REPORT DATE September 1981 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 13. NUMBER OF PAGES 80 |
| | | 15. SECURITY CLASS. (of this report) Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| 18. SUPPLEMENTARY NOTES | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Jackknife Availability Likelihood Log-normal | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Properties of two alternative procedures to the Jackknife Point and Confidence Interval Estimation Procedure of Gaver and Chu have been studied. They are called the Log-Normal Likelihood Procedure (LNLJ) and the Moment Procedure (MP). These two procedures were investigated and compared with the Jackknife Point and Confidence Interval Availability Estimation Procedure. Numerical results from simulations are presented in this report. | | |

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Investigation of Alternative Methods Including Jackknifing
for Estimating Point Availability of a System

by

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Lieutenant, Turkish Navy
B.S., U.S. Naval Postgraduate School, 1981

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September, 1981

ABSTRACT

Properties of two alternative procedures to the Jackknife Point and Confidence Interval Estimation Procedure of Gaver and Chu have been studied. They are called the Log-Normal Likelihood Procedure (LN LJ) and the Moment Procedure (MP). These two procedures were investigated and compared with the Jackknife Point and Confidence Interval Availability Estimation Procedure. Numerical results from simulations are presented in this report.

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ACKNOWLEDGMENT

The author wishes to express his appreciation to the individuals who have given their time in this effort. Specifically, I would like to recognize Professor D. P. Gaver for his guidance, assistance and patience; Professor J. D. Esary, who guided me while Professor D. P. Gaver was out of the U.S.A; Professor R. N. Forrest as the second reader; and my wife, Fügen, for her patience during this study.

I. INTRODUCTION

A. OVERVIEW

Availability is the measure of equipment effectiveness that relates reliability and maintainability to operational readiness. In some cases, availability and operational readiness have been considered the same. In general, these are all requirements which must be considered during the design and operation of a system, and, thus, must be quantitatively evaluated.

There are a variety of ways of expressing availability. In general, availability relates up time (reliability-related) to down time (maintainability-related), and it may be defined as the ratio between the time the system is capable of performing its mission to the total time the system is in operational demand. Alternatively, it is a measure of the probability that a single system is "up" or available at a possibly "random" point in time when its services are needed.

The three expressions of availability of greatest concern are -- (1) inherent availability (A_i), (2) operational

availability (Ao), and (3) achieved availability (Aa). See Riese and Bjorklund[Ref. 1]. In this study, we will be concerned with only the inherent availability. Often, inherent availability, which is a hardware oriented measure, is specified and required within a maintainability contract requirement.

A matter of primary concern is that of estimating availability from observations on system up times and down times. The estimating process should furnish both a point, or single number, estimate and also some measure of the stability of the estimate such as a standard error, or confidence limits. The latter problem is more difficult than the former.

In the paper by Gaver and Chu[Ref. 2], it has been demonstrated that the jackknife technique can be useful for confidence interval estimation of system availability. For instance, let

$$A = \frac{E(U)}{E(U) + E(D)} \quad (1.1)$$

be the long run system availability of a single unit system that changes from up to down states in accordance with a two-state semi-Markov (or more general) process for which up [down] state expected duration is $E(U)$ [$E(D)$]. Then it is

possible to successfully jackknife the naive point estimator that involves replacing expectations by the corresponding sample means,

$$\tilde{A} = \frac{\bar{U}}{\bar{U} + \bar{D}} \quad (1.2)$$

after initial logistic transformation. There are, however, particular parametric families of distributions, popular as models for summarizing up and down time data, for which the most statistically efficient estimators of $E(U)$ and $E(D)$ are not the simple sample arithmetic means. In particular, this is true for,

- (1). The log-normal distribution with unknown (to be estimated) parameters; often this is used as a model for down or repair times. In particular, the log normal distribution has enjoyed considerable popularity for representing electronic system repair times. See Kline and Almog[Ref. 3].
- (2). the general gamma distribution with unknown shape and scale parameters, and
- (3). the general Weibull distribution with unknown shape and scale parameters.

B. PURPOSE AND APPROACH

1. Objectives

Availability estimation by use of the jackknife has already been studied by Gaver and Chu. This thesis presents two procedures alternative to the above. These may be called the Log-normal Likelihood jackknife (LNLJ) procedure and the Moments procedure (MP). In brief, the jackknife method has the capacity to reduce the statistical bias of estimates of such quantities as system availability, and also, and more importantly, to furnish confidence limits that behave in a satisfactory manner (economically enclose the true availability) despite the fact that underlying distributions are unknown. See Miller[Ref. 4] for a review of much of the literature on jackknifing.

The present estimation procedure makes use of the specific assumption of a log normal model for repair times to compute the maximum likelihood estimate (mle) of availability. Properties of the procedure have been studied by Monte Carlo simulation. A number of such simulation results are presented in this thesis. Also presented are comparisons with the direct method of Gaver and Chu, and with a computationally simple method based on moment estimation.

Attention also has been paid to study of the sensitivity of the LNLJ procedure to specification error. This refers to the error introduced by assumption of a log-normal distributional model and the corresponding maximum likelihood estimates when, in fact, another distribution governs the observations. It is found that LNLJ method, which is theoretically most efficient in large samples if the model (likelihood) is correct, can sometimes produce consistently biased results when the log normal model does not apply. Since it is nearly impossible to assure log normality from small samples of data, this finding suggests that caution is in order.

2. Log normal Likelihood Procedure For a Single Unit

(a). Data is given as follows,

$$u_1, u_2, \dots, u_n \quad (\text{up times})$$

$$d_1, d_2, \dots, d_n \quad (\text{down times})$$

$$x_1 = \ln d_1, x_2 = \ln d_2, \dots, x_n = \ln d_n$$

(b). Assume down times are log-normal. Then,

$$E[D] = \exp(\mu_d + 1/2 \cdot \sigma_d^2) \quad (1.2.1)$$

where

$$\mu_d = E[\ln D] \quad (1.2.2)$$

$$\sigma_d^2 = \text{Var}[\ln D] \quad (1.2.3)$$

Furthermore, the maximum likelihood estimates

(MLE) of μ_d and σ_d^2 are

$$\hat{\mu}_d = \bar{x} = 1/n \sum \ln(d_i) \quad (1.2.4)$$

$$\hat{\sigma}_d^2 = 1/n \sum (x_i - \bar{x})^2, \quad (1.2.5)$$

so the MLE of $E(D)$ is, by invariance,

$$\tilde{E}[D] = \exp(\hat{\mu}_d + 1/2 \cdot \hat{\sigma}_d^2) \quad (1.2.6)$$

it is essentially the latter expression that will be used in the estimate for availability to replace the simple first moment estimate $E(D) = \bar{d}$.

In fact, $\hat{\sigma}_d^2$ has been replaced by its unbiased version (n replaced by $n-1$).

(c). If the above holds, it is advantageous to transform first (See Mosteller and Tukey[Ref. 5]) the estimated availability

$$A = \frac{E(U)}{E(U) + E(D)} ;$$

the log-logistic transform of availability is

$$\ln(\tilde{A} / 1 - \tilde{A}) = \ln \tilde{E}(u) - (\hat{\mu}_d + 1/2 \cdot \hat{\sigma}_d^2) \quad (1.2.7)$$

Jackknifing will be carried out using the statistic

$$L_{LN} = \ln(\tilde{A} / 1 - \tilde{A}) = \ln(\bar{u}) - (\bar{x} + 1/2 \cdot s_x^2) \quad (1.2.8)$$

(d) . Recompute L_{LN} repeatedly, leaving out successively the sample pairs (u_1, d_1) , (u_2, d_2) , ..., (u_n, d_n)

$$\bar{u}_{-j} = \frac{1}{n-1} \cdot \left\{ \sum_{i=1}^{j-1} u_i + \sum_{i=j+1}^n u_i \right\} \quad (1.2.9)$$

$$\bar{x}_{-j} = \frac{1}{n-1} \cdot \left\{ \sum_{i=1}^{j-1} x_i + \sum_{i=j+1}^n x_i \right\} \quad (1.2.10)$$

$$s_{x-j}^2 = \frac{1}{n-2} \cdot \left\{ \sum_{i=1}^{j-1} (x_i - \bar{x}_{-j})^2 + \sum_{i=j+1}^n (x_i - \bar{x}_{-j})^2 \right\} \quad (1.1.11)$$

$$L_{LN,j} = \ln(\bar{u}) - (\bar{x}_{-j} + 1/2 \cdot s_{x-j}^2) \quad (1.2.12)$$

where $j=1, 2, \dots, n$

(e) . Compute the pseudovalue as follows:

$$\bar{P}_{LN,j} = n L_{LN,j} - (n-1) L_{LN,-j} \quad j=1, 2, \dots, n \quad (1.2.13)$$

Recall that $\bar{P}_{LN} = \bar{P}_{LN,all}$ is the result of the computing the quantity to be jackknifed, leaving out none of data.

(f) . Compute the mean and variance of the pseudovalue,

$$\bar{P}_{LN} = \frac{1}{n} \cdot \sum_{j=1}^n \bar{P}_{LN,j} \quad (1.2.14)$$

$$S_{LN}^2 = \frac{1}{n-1} \cdot \sum_{j=1}^n (\bar{P}_{LN,j} - \bar{P}_{LN})^2 \quad (1.2.15)$$

(g) . The jackknifed point estimate of the availability is now,

$$A = \exp(\bar{P}_{LN}) / (1 + \exp(\bar{P}_{LN})) \quad (1.2.16)$$

(h) . Symmetric two sided confidence limits at confidence level $(1-\alpha)^{\frac{1}{2}}$ are derived as follows;

$$CL_{LN} = \bar{P}_{LN} - t_{1-\alpha/2} (n-1) \cdot \text{SQRT}(1/n \cdot S_{LN}^2) \quad (1.2.17)$$

$$CU_{LN} = \bar{P}_{LN} + t_{1-\alpha/2}(n-1) \sqrt{1/n \cdot S_{LN}^2} \quad (1.2.18)$$

where $t_{1-\alpha/2}(n-1)$ is the $(1-\alpha/2)100\%$ quantile of Student's-t with $n-1$ degrees of freedom. Then,

$$\frac{\exp(CL_{LN})}{1 + \exp(CL_{LN})} \leq A \leq \frac{\exp(CU_{LN})}{1 + \exp(CU_{LN})} \quad (1.2.19)$$

with confidence approximately $(1-\alpha)100\%$. Note that the confidence limits are nearly symmetric around $\ln(E[U]/E[D])$, and not around A.

(i). One sided confidence limits at confidence level $(1-\alpha)100\%$ are derived

$$CL_{LN} = \bar{P}_{LN} - t_{1-\alpha}(n-1) \sqrt{1/n \cdot S_{LN}^2} \quad (1.2.20)$$

$$CU_{LN} = \bar{P}_{LN} + t_{1-\alpha}(n-1) \sqrt{1/n \cdot S_{LN}^2} \quad (1.2.21)$$

So, one-sided upper confidence limit is

$$A \leq \frac{\exp(CU_{LN})}{1 + \exp(CU_{LN})} \quad (1.2.22)$$

and lower confidence limit is

$$A \leq \frac{\exp(CL_{LN})}{1 + \exp(CL_{LN})} \quad (1.2.23)$$

Note that if up times are exponentially distributed then u is actually the mle of $E[U]$, so under the model assumptions the (transformed) mle of availability is being

jackknifed; this procedure has been validated by Reeds [Ref. 6] for large samples. Simulation studies are nevertheless essential for investigating properties of the jackknife or alternative techniques for small samples, and for investigating sensitivity to model assumption.

3. Moment Procedure For a Single Unit

(a). Data given as follows

$$u_1, u_2, \dots, u_n \quad (\text{up times})$$

$$d_1, d_2, \dots, d_n \quad (\text{down times})$$

$$y_1 = \ln u_1, y_2 = \ln u_2, \dots, y_n = \ln u_n$$

$$x_1 = \ln d_1, x_2 = \ln d_2, \dots, x_n = \ln d_n$$

(b). Assume up and down times are log-normal. Then,

$$E[U] = \exp(\mu_u + 1/2 \cdot \sigma_u^2) \quad (1.3.1)$$

$$E[D] = \exp(\mu_d + 1/2 \cdot \sigma_d^2) \quad (1.3.2)$$

where,

$$\mu_u = E[\ln U] \quad (1.3.3)$$

$$\mu_d = E[\ln D] \quad (1.3.4)$$

$$\sigma_u^2 = \text{Var}[\ln U] \quad (1.3.5)$$

$$\sigma_d^2 = \text{Var}[\ln D] \quad (1.3.6)$$

Furthermore, the mle of μ_u , σ_u^2 , μ_d , and σ_d^2 are

$$\hat{\mu}_u = \bar{y} = 1/n \cdot \sum_{i=1}^n y_i \quad (1.3.7)$$

$$\hat{\sigma}_u^2 = 1/n \cdot \sum_{i=1}^n (y_i - \bar{y})^2 \quad (1.3.8)$$

$$\hat{\mu}_d = \bar{x} = 1/n \cdot \sum_{i=1}^n x_i \quad (1.3.9)$$

$$\tilde{\sigma}_d^2 = 1/n \cdot \sum_{i=1}^n (x_i - \bar{x})^2 \quad (1.3.10)$$

so the mle of $E[U]$ and $E[D]$ are, by invariance,

$$E[U] = \exp(\tilde{\mu}_u + 1/2 \cdot \tilde{\sigma}_u^2) \quad (1.3.11)$$

$$E[D] = \exp(\tilde{\mu}_d + 1/2 \cdot \tilde{\sigma}_d^2) \quad (1.3.12)$$

- (c) If above holds, the logistic transform of availability is,

$$\ln(\tilde{A} / (1-\tilde{A})) = (\tilde{\mu}_u + 1/2 \cdot \tilde{\sigma}_u^2) - (\tilde{\mu}_d + 1/2 \cdot \tilde{\sigma}_d^2) \quad (1.3.13)$$

estimation will be carried out using the statistic

$$P = \hat{\ln}(A / (1-A)) = (\bar{y} + 1/2 \cdot s_y^2) - (\bar{x} + 1/2 \cdot s_x^2) \quad (1.3.14)$$

- (d) Simply the point estimate of availability is now,

$$A = \exp(P) / 1 + \exp(P) \quad (1.3.15)$$

- (e) Variance of the statistic P can be calculated as follows,

$$\text{Var}(P) = (SE)^2 = 1/n \cdot s_y^2 + 1/4 \cdot \text{Var}[s_y^2] + 1/n \cdot s_x^2 + 1/4 \cdot \text{Var}[s_x^2] \quad (1.3.16)$$

where,

$$\text{Var}[s_y^2] = 1/n [M_{4y} - (s_y^2)^2] \quad (1.3.17)$$

$$\text{Var}[s_x^2] = 1/n [M_{4x} - (s_x^2)^2] \quad (1.3.18)$$

- (f) Symmetric two sided confidence limits at confidence level $(1-\alpha)\%$ are derived as follows,

$$L = P - z_{1-\alpha/2} (SE)^2 \quad (1.3.19)$$

$$U = p + z_{1-\alpha/2} (SE)^2 \quad (1.3.20)$$

where $z_{1-\alpha/2}$ is the $(1 - \alpha/2) 100\%$ quantile of the standard normal distribution. Then,

$$\{\exp(L) / 1 + \exp(L)\} \leq A \leq \{\exp(U) / 1 + \exp(U)\} \quad , \quad (1.3.21)$$

II. SIMULATION PROCEDURE

A simulation procedure has been used to compare the operating characteristics of LNLJ methodology introduced here with other approaches. Specifically, simulation has been used to compute

- (a) the actual coverage of the true availability figure, λ , by the confidence intervals given by the procedures under study, when the nominal coverage is $(1-\alpha)100\%$,
- (b) measures of confidence interval expected width and variance width,
- (c) estimate of the expected point availability estimated by the procedure under study.

The simulation program was written in FORTRAN IV, and the simulation have been carried out on the IBM 3033 at the Naval Postgraduate School. A more detailed description of the program and its major subroutines is given in Appendix A.

An outline of the simulation procedure now follows:

- (A) A simulated sample of up-time durations, $(u_i, i=1, 2, \dots, n)$ and down-time durations, $(d_i, i=1, 2, \dots, n)$ were

times. The Naval Postgraduate School LLRANDOM package was used, along with the International Mathematical and Statistical Library (IMSL) random number generator. For all cases considered (exceptions noted), expected up and down times were $E[U]=73.036$ and $E[D]=3.844231$ for the population sampled, so long-run availability $A=0.95$. Sample sizes of $n=15$ and $n=25$ were simulated, and provided the bases for point estimators and for confidence intervals.

(B) Here are the different distributional situations that were sampled. A total of 1000 replications was used to evaluate each procedure in each distributional situation.

- (1) Up-times come independently from the iid $\text{Expon}(\lambda)$ distributions. Down-times come independently from the log-normal (μ, σ^2) distribution.
- (2) Up-times are iid $\text{Expon}(\lambda)$. Down times are iid $\text{Expon}(\mu)$.
- (3) Up-times are iid $\text{Expon}(\lambda)$. Down-times are iid $\text{Gamma}(\theta, k)$, being a scale and k a shape parameter, so arranged that $E[D] = \mu^{-1}$ and $\text{Var}[D] = (\sqrt{k}\mu)^{-2}$. A Gamma with $k > 1$, e.g. $k=2$, represents data that is more tightly grouped around its mean than is true of exponentially distributed data; it roughly resembles

data that is log-normal. A gamma with $k < 1$, e.g. $k = 1/2$, represents data of relatively extreme dispersion and positive skewness as compared to exponential data having a very long right tail, compensated for by a high density near zero. A gamma distribution with integer k ($k = 1, 2, 3, \dots$) is often called Erlang; realizations are easily simulated by summing k independent Exponential realizations.

- (4) Up-times are iid $\text{Expon}(\lambda)$. Down-times are iid long-tailed h distribution:

$$d = E[D] \cdot w \cdot \exp(h \cdot w) \cdot (1-h)^2, \quad 0 < h < 1,$$

where w comes from $\text{Expon}(1)$. This long-tailed h distribution possesses no closed-form representation. However, it can be shown to have characteristics similar to those of the Exponential for small values of h , but to have a systematically longer right tail than the exponential.

- (5) Up times are iid long-tailed h distribution. Down times are iid $\text{Expon}(\mu)$.

- (6) Up times are iid $\text{Expon}(\lambda)$. Down times are iid Weibull(θ, k), being a scale parameter and k a shape parameter, so arranged that $E[D] = \theta^{1/k} \cdot \Gamma(1/k + 1)$ and

$\text{Var}[D] = \theta^{2/k} \cdot [\Gamma(2/k + 1) - \Gamma^2(1/k + 1)]$. A Weibull with $k > 1$, e.g. $k=2$, represents data that is more tightly grouped around its mean than is true of exponentially distributed data; it quantitatively resembles data that is log-normal. A Weibull with $k < 1$, e.g. $k=1/2$, represents data of relatively extreme dispersion and positive skewness as compared to exponential data a very long right tail, compensated for by a high density near zero.

- (7) Up times are iid $\text{Expon}(\lambda)$. Down times are iid long-tailed log-normal h distribution. See Appendix B.

III. ANALYSIS

The methods described were simulated for various distributional assumptions which were defined in Chapter 2. Simulation results for each method and set of distributional assumptions are shown in Table 1 and Table 2.

In general, MP has very high coverage factor and satisfactory point availability estimation. However, both average and variance of confidence length are higher than JK and LNLJ procedures. In addition, no consistent superiority of LNLJ procedure over JK procedure has been noted.

Especially under the Exponential up and Gamma ($k=1/2$) down, and Exponential up and Weibull ($k=1/2$) down times assumptions, a consistently low coverage factor and point availability estimation, and a consistently high average confidence length and variation have been obtained from LNLJ procedure. In these cases, distributional characteristics of down times after log-transform provide highly left skewed shape and large variance, and, also these features reflect in the pseudovalues which, in turn, causes left skewed shape rather than a symmetric shape for the pseudovalue distribution.

In order to compensate for this situation, the Biweight procedure, see[Ref. 7], and the Winsorizing procedure, see [Ref. 8], were implemented directly on the pseudovalue s. Results are shown in Tables 3, 4, 5, 6, 7, 8. These procedures are robust/ resistant methods for establishing confidence limits on means; they tend to down-weight extreme observations that appears as outliers.

For small sample size, (i.e. n=15) Winsorizing at g=2 and g=3 levels provide an adequate coverage. But, the point availability estimate and average confidence length remain inadequate. In general, the Biweight statistical procedure caused reduction of the coverage factor, and it is not yet considered effective for compensation.

Next, a grouped jackknife procedure was applied to these cases. Simply, up and down times data were grouped two by two, and average of these groups have been taken. Obviously, this process reduced the sample sizes to one half and caused loss of degrees of freedom. However, smaller variation and more stable results were anticipated. Simulation results are presented in Tables 9 and 10. Very good coverage factors and point availability estimates were obtained, but, average confidence length remained at a rather high level. See

Appendix C for histograms of both cases for up and down times, and log-transform of down times and pseudovales after grouped jackknife procedure applied.

Other critical simulation results were obtained from exponential up and long-tailed log-normal h down times case. For both JK and LNLJ procedures, above statistical procedures were applied, and results were shown in Tables 11, 12, 13, 14, 15, 16. In either case, after implementation of these statistical representations, the coverage factor decreased. Also, see Appendix C for histograms of this case.

Table 1: Simulation Results for Distributional Assumptions

| Sample size=15 | | | | | | |
|----------------|----------------------------|----|---------------------|-------------------|----------------|-----------------------|
| | Underlying Distribution | | Average Coverage | Variance Width | Point Width | Point Availability |
| A. | Exponential up | JK | 0.9380 | 0.0838 | 0.0022 | 0.9458 |
| | Log-normal down | LN | 0.9443 | 0.0837 | 0.0016 | 0.9459 |
| | | MP | 0.9698 | 0.0959 | 0.0026 | 0.9556 |
| B. | Exponential up | JK | 0.9470 | 0.0850 | 0.0013 | 0.9471 |
| | Exponential down | LN | 0.9417 | 0.1378 | 0.0128 | 0.9312 |
| | | MP | 0.9868 | 0.1765 | 0.0196 | 0.9421 |
| C. | Exponential up | JK | 0.9423 | 0.0706 | 0.0007 | 0.9465 |
| | Gamma down | LN | 0.9419 | 0.0764 | 0.0010 | 0.9437 |
| | k=2 | MP | 0.9830 | 0.1001 | 0.0031 | 0.9538 |
| D. | Ditto | JK | 0.9450 | 0.1131 | 0.0039 | 0.9442 |
| | k=1/2 | LN | 0.9161 | 0.4574 | 0.0932 | 0.8064 |
| | | MP | 0.9873 | 0.5013 | 0.0970 | 0.8311 |
| E. | Exponential up | JK | 0.9256 | 0.1171 | 0.0097 | 0.9437 |
| | Long-tailed h | LN | 0.9483 | 0.1750 | 0.0197 | 0.9287 |
| | h=0.2 | MP | 0.9780 | 0.1951 | 0.0244 | 0.9412 |
| F. | Ditto | JK | 0.8753 | 0.1663 | 0.0343 | 0.9387 |
| | h=0.4 | LN | 0.9406 | 0.2144 | 0.0342 | 0.9314 |
| | | MP | 0.9513 | 0.2092 | 0.0321 | 0.9431 |

| | | | | | | |
|----|------------------|----|--------|--------|--------|--------|
| G. | Long-tailed h up | JK | 0.9245 | 0.1077 | 0.0024 | 0.9428 |
| | Exponential down | LN | 0.9058 | 0.1650 | 0.0150 | 0.9256 |
| | h=0.2 | MP | 0.9803 | 0.1973 | 0.0212 | 0.9381 |
| H. | Ditto | JK | 0.8837 | 0.1470 | 0.0046 | 0.9351 |
| | h=0.4 | LN | 0.8552 | 0.2100 | 0.0179 | 0.9163 |
| | | MP | 0.9437 | 0.2377 | 0.0244 | 0.9284 |
| I. | Exponential up | JK | 0.9476 | 0.0641 | 0.0005 | 0.9483 |
| | Weibull down | LN | 0.9449 | 0.0672 | 0.0007 | 0.9463 |
| | k=2 | MP | 0.9838 | 0.0917 | 0.0023 | 0.9501 |
| J. | Ditto | JK | 0.9142 | 0.1764 | 0.0239 | 0.9404 |
| | k=1/2 | LN | 0.9336 | 0.5752 | 0.0921 | 0.7649 |
| | | MP | 0.9751 | 0.5819 | 0.0966 | 0.8464 |
| K. | Exponential up | JK | 0.8968 | 0.1385 | 0.0220 | 0.9416 |
| | Long-tailed log | LN | 0.9119 | 0.1217 | 0.0091 | 0.9483 |
| | normal h down | MP | 0.9183 | 0.1113 | 0.0058 | 0.9576 |
| | h=0.2 | | | | | |
| L. | Ditto | JK | 0.8794 | 0.1578 | 0.0305 | 0.9394 |
| | h=0.4 | LN | 0.9009 | 0.1394 | 0.0136 | 0.9483 |
| | | MP | 0.9001 | 0.1210 | 0.0079 | 0.9575 |

Table 2: Simulation Results for Distributional Assumptions

| | | Sample size=25 | | | |
|----------------------------|----|---------------------|-------------------|----------------|-----------------------|
| Underlying Distribution | | Average Coverage | Variance Width | Point Width | Point Availability |
| A. Exponential up | JK | 0.9436 | 0.0594 | 0.0007 | 0.9470 |
| Log-normal down | LN | 0.9473 | 0.0589 | 0.0004 | 0.9471 |
| | MP | 0.9740 | 0.0713 | 0.0008 | 0.9571 |
| B. Exponential up | JK | 0.9473 | 0.0597 | 0.0004 | 0.9471 |
| Exponential down | LN | 0.9228 | 0.0978 | 0.0052 | 0.9322 |
| | MP | 0.9945 | 0.1302 | 0.0086 | 0.9450 |
| C. Exponential up | JK | 0.9475 | 0.0506 | 0.0002 | 0.9474 |
| Gamma down | LN | 0.9414 | 0.0547 | 0.0003 | 0.9447 |
| k=2 | MP | 0.9895 | 0.0751 | 0.0010 | 0.9552 |
| D. Ditto | JK | 0.9464 | 0.0766 | 0.0009 | 0.9463 |
| k=1/2 | LN | 0.8187 | 0.4008 | 0.0790 | 0.8168 |
| | MP | 0.9884 | 0.4606 | 0.0841 | 0.8435 |
| E. Exponential up | JK | 0.9313 | 0.0834 | 0.0041 | 0.9456 |
| Long-tailed h | LN | 0.9464 | 0.1213 | 0.0077 | 0.9323 |
| h=0.2 | MP | 0.9867 | 0.1426 | 0.0108 | 0.9450 |
| F. Ditto | JK | 0.8784 | 0.1261 | 0.0212 | 0.9421 |
| h=0.4 | LN | 0.9475 | 0.1464 | 0.0132 | 0.9366 |
| | MP | 0.9593 | 0.1510 | 0.0139 | 0.9481 |

| | | | | | | |
|----|------------------|----|--------|--------|--------|--------|
| G. | Long-tailed h up | JK | 0.9297 | 0.0759 | 0.0007 | 0.9454 |
| | Exponential down | LN | 0.8901 | 0.1167 | 0.0066 | 0.9300 |
| | h=0.2 | MP | 0.9834 | 0.1447 | 0.0099 | 0.9422 |
| H. | Ditto | JK | 0.8802 | 0.1058 | 0.0017 | 0.9394 |
| | h=0.4 | LN | 0.8248 | 0.1521 | 0.0073 | 0.9228 |
| | | MP | 0.9575 | 0.1771 | 0.0116 | 0.9336 |
| I. | Exponential up | JK | 0.9448 | 0.0464 | 0.0002 | 0.9489 |
| | Weibull down | LN | 0.9433 | 0.0485 | 0.0002 | 0.9470 |
| | k=2 | MP | 0.9879 | 0.0691 | 0.0008 | 0.9494 |
| J. | Ditto | JK | 0.9222 | 0.1188 | 0.0087 | 0.9446 |
| | k=1/2 | LN | 0.8456 | 0.5116 | 0.0796 | 0.7765 |
| | | MP | 0.9727 | 0.5461 | 0.0854 | 0.8245 |
| K. | Exponential up | JK | 0.9010 | 0.1032 | 0.0123 | 0.9440 |
| | Long-tailed log | LN | 0.9141 | 0.0817 | 0.0026 | 0.9507 |
| | normal h down | MP | 0.9078 | 0.0792 | 0.0018 | 0.9600 |
| | h=0.2 | | | | | |
| L. | Ditto | JK | 0.8835 | 0.1184 | 0.0183 | 0.9426 |
| | h=0.4 | LN | 0.9003 | 0.0918 | 0.0037 | 0.9511 |
| | | MP | 0.8930 | 0.0848 | 0.0024 | 0.9604 |

Table 3: Implementation of "Winsorizing" on Exponential Up and Gamma ($k=1/2$) Down Times for LNLJ Procedure
 Sample size=15

| | Coverage | Average Width | Variance Width | Point Availability |
|-----|----------|---------------|----------------|--------------------|
| G=0 | 0.9103 | 0.4212 | 0.0809 | 0.8239 |
| G=1 | 0.9095 | 0.3624 | 0.0572 | 0.8686 |
| G=2 | 0.9272 | 0.3467 | 0.0567 | 0.8840 |
| G=3 | 0.9400 | 0.3427 | 0.0609 | 0.8954 |

Table 4: Implementation of "Winsorizing" on Exponential Up and Gamma ($k=1/2$) Down times for LNLJ Procedure

| | Coverage | Average Width | Variance Width | Point Availability |
|-----|----------|---------------|----------------|--------------------|
| G=0 | 0.8256 | 0.3691 | 0.0670 | 0.8319 |
| G=2 | 0.8010 | 0.2565 | 0.0265 | 0.8829 |
| G=3 | 0.8361 | 0.2456 | 0.0247 | 0.8908 |
| G=4 | 0.8717 | 0.2391 | 0.0247 | 0.8972 |

Table 5: Implementation of "Winsorizing" on Exponential Up and Weibull ($k=1/2$) Down Times for LNLJ Procedure
 Sample size=15

| | Coverage | Average | Variance | Point Availability |
|-----|----------|---------|----------|--------------------|
| | | Width | Width | |
| G=0 | 0.9336 | 0.5752 | 0.0921 | 0.7649 |
| G=1 | 0.9418 | 0.5056 | 0.0828 | 0.8323 |
| G=2 | 0.9526 | 0.4699 | 0.0855 | 0.8611 |
| G=3 | 0.9510 | 0.4464 | 0.0907 | 0.8836 |

Table 6: Implementation of "Winsorizing" on Exponential Up and Weibull ($k=1/2$) Down Times for LNLJ Procedure
 Sample size=25

| | Coverage | Average | Variance | Point Availability |
|-----|----------|---------|----------|--------------------|
| | | Width | Width | |
| G=0 | 0.8456 | 0.5116 | 0.0796 | 0.7765 |
| G=2 | 0.8692 | 0.3874 | 0.0516 | 0.8537 |
| G=3 | 0.9051 | 0.3632 | 0.0513 | 0.8689 |
| G=4 | 0.9316 | 0.3431 | 0.0511 | 0.8829 |

Table 7: Implementation of "Biweighting" on Exponential Up and Gamma ($k=1/2$) Down Times for LNLJ Procedure

| | Sample size | Average Coverage | Variance width | Point width | Availability |
|------------|----------------|---------------------|-------------------|----------------|--------------|
| Original | 15 | 0.9103 | 0.4212 | 0.0809 | 0.8223 |
| Biweighted | 15 | 0.8935 | 0.2621 | 0.0448 | 0.9117 |
| Original | 25 | 0.8256 | 0.3691 | 0.0670 | 0.8319 |
| Biweighted | 25 | 0.8786 | 0.1768 | 0.0159 | 0.9195 |

Table 8: Implementation of "Biweighting" on Exponential Up and Weibull ($k=1/2$) Down Times for LNLJ Procedure

| | Sample size | Average Coverage | Variance width | Point width | Availability |
|------------|----------------|---------------------|-------------------|----------------|--------------|
| Original | 15 | 0.9336 | 0.5752 | 0.0921 | 0.7649 |
| Biweighted | 15 | 0.8423 | 0.3163 | 0.0784 | 0.9071 |
| Original | 25 | 0.8456 | 0.5116 | 0.0796 | 0.7765 |
| Biweighted | 25 | 0.8374 | 0.2134 | 0.0362 | 0.9231 |

Table 9: Implementation of "Grouped Jackknife" Procedure for Exponential Up and Weibull ($k=1/2$) Down Times

| Method | Sample Size | Average | Variance | Point Availability | |
|--------|-------------|----------|----------|--------------------|--------|
| | | Coverage | Width | Width | |
| JK | 14 | 0.9209 | 0.2136 | 0.0353 | 0.9381 |
| LN | 14 | 0.9486 | 0.3648 | 0.0774 | 0.9038 |
| MP | 14 | 0.9390 | 0.3083 | 0.0672 | 0.9073 |
| JK | 24 | 0.9223 | 0.1319 | 0.0118 | 0.9429 |
| LN | 24 | 0.9500 | 0.2469 | 0.0400 | 0.9148 |
| MP | 24 | 0.9685 | 0.2518 | 0.0452 | 0.9183 |

Table 10: Implementation of "Grouped Jackknife" Procedure for Exponential Up and Gamma ($k=1/2$) Down Times

| Method | Sample Size | Average | Variance | Point Availability | |
|--------|-------------|----------|----------|--------------------|--------|
| | | Coverage | Width | Width | |
| JK | 14 | 0.9426 | 0.1380 | 0.0080 | 0.9441 |
| LN | 14 | 0.9534 | 0.2266 | 0.0381 | 0.9243 |
| MP | 14 | 0.9608 | 0.2186 | 0.0419 | 0.9262 |
| JK | 24 | 0.9472 | 0.0839 | 0.0015 | 0.9461 |
| LN | 24 | 0.9462 | 0.1448 | 0.0174 | 0.9293 |
| MP | 24 | 0.9783 | 0.1710 | 0.0259 | 0.9322 |

Table 11: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal ($h=0.2$) Down Times

| Sample size=15 | | | | |
|----------------|---------|----------|--------------|--------|
| | Average | Variance | Point | |
| Coverage | Width | Width | Availability | |
| JK | | | | |
| G=0 | 0.8968 | 0.1385 | 0.0220 | 0.9416 |
| G=1 | 0.8190 | 0.0700 | 0.0019 | 0.9544 |
| G=2 | 0.7450 | 0.0585 | 0.0011 | 0.9560 |
| G=3 | 0.7160 | 0.054 | 0.0009 | 0.9565 |
| LN | | | | |
| G=0 | 0.9119 | 0.1217 | 0.0091 | 0.9483 |
| G=1 | 0.8560 | 0.0898 | 0.0040 | 0.9549 |
| G=2 | 0.8040 | 0.0759 | 0.0029 | 0.9575 |
| G=3 | 0.7460 | 0.0668 | 0.0027 | 0.9598 |

Table 12: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal ($h=0.2$) Down Times

| | Sample size=25 | | | |
|-----------|----------------|----------|--------|--------------|
| | Average | Variance | Point | |
| | Coverage | Width | Width | Availability |
| JK | | | | |
| G=0 | 0.9010 | 0.1032 | 0.0123 | 0.9440 |
| G=2 | 0.7380 | 0.0460 | 0.0004 | 0.9554 |
| G=3 | 0.6950 | 0.0416 | 0.0001 | 0.9563 |
| G=4 | 0.6620 | 0.0387 | 0.0003 | 0.9568 |
| LN | | | | |
| G=0 | 0.9141 | 0.0817 | 0.0026 | 0.9507 |
| G=2 | 0.8240 | 0.0594 | 0.0010 | 0.9558 |
| G=3 | 0.7750 | 0.0539 | 0.0008 | 0.9573 |
| G=4 | 0.7380 | 0.0488 | 0.0006 | 0.9588 |

Table 13: Implementation of "Winsorizing" on Exponential Up
and Long-Tailed Log-Normal ($h=0.4$) Down Times

| | Sample size=15 | | | |
|-----------|----------------|----------|--------|--------------|
| | Average | Variance | Point | |
| | Coverage | Width | Width | Availability |
| JK | | | | |
| G=0 | 0.8794 | 0.1578 | 0.0305 | 0.9394 |
| G=1 | 0.7560 | 0.0730 | 0.0026 | 0.9542 |
| G=2 | 0.6870 | 0.0588 | 0.0015 | 0.9560 |
| G=3 | 0.6460 | 0.0537 | 0.0012 | 0.9565 |
| LN | | | | |
| G=0 | 0.9009 | 0.1394 | 0.0136 | 0.9483 |
| G=1 | 0.8470 | 0.1049 | 0.0069 | 0.9543 |
| G=2 | 0.7800 | 0.0866 | 0.0053 | 0.9576 |
| G=3 | 0.7180 | 0.0763 | 0.0048 | 0.9603 |

Table 14: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal ($h=0.4$) Down Times

| | Sample size=25 | | | |
|-----------|----------------|---------------|----------------|--------------------|
| | Coverage | Average Width | Variance Width | Point Availability |
| JK | | | | |
| G=0 | 0.8835 | 0.1184 | 0.0183 | 0.9426 |
| G=2 | 0.6810 | 0.0473 | 0.0007 | 0.9560 |
| G=3 | 0.6220 | 0.0411 | 0.0004 | 0.9572 |
| G=4 | 0.5750 | 0.0380 | 0.0004 | 0.9578 |
| LN | | | | |
| G=0 | 0.9003 | 0.0918 | 0.0037 | 0.9511 |
| G=2 | 0.8000 | 0.0666 | 0.0018 | 0.9568 |
| G=3 | 0.7660 | 0.0585 | 0.0014 | 0.9590 |
| G=4 | 0.7000 | 0.0531 | 0.0012 | 0.9607 |

Table 15: Implementation of "Biweighting" on Exponential Up and Long-Tailed Log-Normal ($h=0.2$) Down Times

| | Sample size | Average Coverage | Variance Width | Point width | Point Availability |
|------------|----------------|---------------------|-------------------|----------------|-----------------------|
| JK | | | | | |
| Original | 15 | 0.8968 | 0.1385 | 0.0220 | 0.9416 |
| Biweighted | 15 | 0.6870 | 0.0528 | 0.0010 | 0.9554 |
| Original | 25 | 0.9010 | 0.1032 | 0.0123 | 0.9440 |
| Biweighted | 25 | 0.6360 | 0.0362 | 0.0002 | 0.9569 |
| LN | | | | | |
| Original | 15 | 0.9119 | 0.1217 | 0.0091 | 0.9483 |
| Biweighted | 15 | 0.7010 | 0.0604 | 0.0019 | 0.9607 |
| Original | 25 | 0.9141 | 0.0817 | 0.0026 | 0.9507 |
| Biweighted | 25 | 0.6170 | 0.0406 | 0.0004 | 0.9627 |

Table 16: Implementation of "Biweighting" on Exponential Up and Long-Tailed Log-Normal ($h=0.4$) Down Times

| | Sample size | Average Coverage | Average width | Variance width | Point Availability |
|------------|-------------|------------------|---------------|----------------|--------------------|
| JK | | | | | |
| Original | 15 | 0.8794 | 0.1578 | 0.0305 | 0.9394 |
| Biweighted | 15 | 0.6210 | 0.0499 | 0.0010 | 0.9578 |
| Original | 25 | 0.8835 | 0.1184 | 0.0183 | 0.9426 |
| Biweighted | 25 | 0.5500 | 0.0340 | 0.0002 | 0.9590 |
| LN | | | | | |
| Original | 15 | 0.9009 | 0.1394 | 0.0136 | 0.9483 |
| Biweighted | 15 | 0.6350 | 0.0614 | 0.0025 | 0.9642 |
| Original | 25 | 0.9003 | 0.0918 | 0.0037 | 0.9511 |
| Biweighted | 25 | 0.5261 | 0.0394 | 0.0005 | 0.9663 |

IV. CONCLUSIONS

The LNLJ procedure works well when the actual down times are Exponential or Log-normal. However, the procedure is very sensitive to the sample variance. If the sample variance tends to vary greatly, sometimes becoming excessively large, then the method tends to fail; the intervals shifted down. For instance, long tailed Gamma, Weibull and Log-normal distributed down times provide extremely large sample variances, and under these circumstances the method fails.

For further study, one might apply jackknifing leaving out one observation at a time, instead leaving out a pair. Obviously, this is going to increase the degrees of freedom, but might by help reducing the variance and the confidence interval length. On the other hand, grouping more than two, might provide more stable results.

In addition, in order to see what happens in one-sided availability estimation, Exponential up and Log-normal down and Exponential up Weibull ($k=1/2$) down times were simulated and results are shown in Appendix D.

APPENDIX A

COMPUTER PROGRAMS

Simulation program consists of main program and several subroutines. Main program, for JK, LNLJ and MP procedures computes availability confidence limits and point availability and, scores the coverage for each replication. Then, after 1000 replications computes the statistics of these parameters and prints out the results for given underlying distributions.

Subroutine CONF computes two-sided confidence limits of a given data vector. Subroutine MOMENT computes the fourth moment of given data vector. Subroutine LNLJ generates the up and down times from given underlying distributions, and takes the log transform of these, and computes the means and the variances of these data vectors.

Subroutine BIWGT computes the Biweight estimates of a given data vector, and uses Subroutine MEDIAN for computing the median of this data. Subroutine WINSOR Winsorize a given data vector for given "g" level.


```

***** VARIABLE DEFINITION *****
U = UP TIMES VECTOR
D = DOWN( REPAIR ) TIMES VECTOR
LOG = LOG TRANSFORM OF DOWN TIMES
X = LOG TRANSFORM OF UP TIMES
Y = SAMPLE MEAN OF UP TIMES
DBAR = " " DOWN
XBAR = " " " X VALUES
YBAR = " " " Y VALUES
XVAR = " VARIANCE OF X VALUES
YVAR = " VARIANCE OF Y VALUES
UMEAN=UP TIMES POPULATION MEAN
DMEAN=DOWN TIMES POPULATION MEAN
ALPHA=TYPE I ERROR
CNF = CONFIDENCE

```

JK PROCEDURE VARIABLES

```

LS = LOG(A/(1-A))=LOG(UBAR)-LOG(DBAR)
LSJ = J-TH VERSION OF LS
PS = PSEUDOVALUES
PSBAR=MEAN OF PSEUDOVALUES
S2S = VARIANCE OF " " " LOWER BOUND FOR PSEUDOVALUES
CLS = UPPER " "
CUS = LOWER JACKKNIFE AVAILABILITY ESTIMATE
AJEA = JACKKNIFE AVAILABILITY ESTIMATE
ALS = LOWER JACKKNIFE AVAILABILITY
AUS = UPPER DELTA=CONFIDENCE LENGTH OF AVAILABILITY
DELTA=AVERAGE OF DELTA
AVG = AVERAGE OF DELTA
VAR = VARIANCE OF DELTA
SCORE=COUNTER FOR SUCCESSFUL TRIALS
COVER=PERCENTAGE OF SUCCESSFUL TRIALS

```

LNJ PROCEDURE VARIABLES

```

LLN = LOG(A/(1-A))=LOG(UBAR)-(XBAR+0.5*S2X)
XX = MEAN OF X VECTOR OMITTING J-TH VALUE
SSX = VARIANCE OF X VECTOR OMITTING J-TH VALUE
LLNJ=J-TH VERSION OF LLN
PLN = PSEUDOVALUES
PLNBAR=MEAN OF PSEUDOVALUES
S2LN=VARIANCE OF " "
CLLN=LOWER BOUND FOR PSEUDOVALUES
CULN=UPPER AJEB= JACKKNIFE AVAILABILITY ESTIMATE

```



```

C**          ALLN=LOWER   "    "    AVAILABILITY
C**          AULN=UPPER  "    "    "
C***          TEMPORARY VARIABLES; TEMP,A,B,C,C1,C2,SUM,SUMX,Z,ZZ
C***          DIMENSION U(25),D(25),X(25),Y(25),PS(25),PLN(25),DELTAI(1000),
C***          DELTA2(1000),DELTAI3(1000)
C***          INTEGER N,RUN,IS1,IS2,I,J,K,L,IS11,IS22
C***          REAL Q1,Q2,Q3,Q4,Q5,Q6,Z,U,Y,X,D,UBAR,UMEAN,DMEAN,ALPHA,
C***          DBAR,XBAR,YBAR,LS,A,B,LSJ,PS,PSBAR,S2S,LLN,C,C1,SUM,SUMX,
C***          XX,C2,SSX,LLNJ,PLN,PLNBAR,SCUS,ALS,AUS,DELTA1,ALLN,
C***          SCORE1,AULN,C,LLN,DELTAA2,SCORE2,COVER1,COVER2,AVG1,AVG2,CNF,
C***          STD1,STD2,CULN,TEMP1,TEMP2,TEMP3,TEMP4,TEMP5,TEMP6,TEMP7,
C***          XVAR,YVAR,XM4,YM4,AVG3,DELTA3,VAR3,SESEQ,PM,H
C***          UMEAN=73.036
C***          XMU=1.
C***          XSD=0.8325546
C***          H=0.4
C***          ALPHA=0.05
C***          CNF=1.-ALPHA
C***          L=0
C***          N=15
C***          RUN=1000
C***          GO TO 20
C***          N=25
C***          IS1=100660
C***          IS2=100010
C***          L=L+1
C***          Q1=0.
C***          Q2=0.
C***          Q3=0.
C***          Q4=0.
C***          Q5=0.
C***          Q6=0.
C***          TEMP6=0.
C***          TEMP7=0.
C***          TEMP8=0.
C***          DO 100 I=1, RUN
C***          CALL LN(J((IS1,IS2,XMU,XSD,UMEAN,H,U,Y,D,X,UBAR,YBAR,TEMP1,TEMP2,TEMP3,IS11,IS22),
C***          IS1=IS11
C***          IS2=IS22
C***          COMPUTE LS AND PSEUDOVALUES
C***          LS=ANALOG(UBAR)-ANALOG(DBAR)
C***          A=TEMP1
C***          B=TEMP2

```



```

TEMP4=0.
DO 103 J=1,N
    TEMP1=TEMP1-U(J)
    TEMP2=TEMP2-D(J)
    LSJ=ALOG(TEMP1)-ALOG(TEMP2)
    PS(J)=(LS*N)-(LSJ*(N-1))
    TEMP4=TEMP4+PS(J)
    TEMP1=A
    TEMP2=B
CONTINUE
PSBAR=TEMP4/N
TEMP4=0.
DO 104 J=1,N
    TEMP4=TEMP4+(PS(J)-PSBAR)**2
CONTINUE
S2S=TEMP4/(N-1)
AJEA=(EXP(PSBAR))/(1.+EXP(PSBAR))
TEMP6=TEMP6+AJEA

C COMPUTE LLN AND PSEUDOVALUES
C=TEMP3
LLN=ALOG(UBAR)-(XBAR+0.5*XVAR)
SUM=0.
DO 105 J=1,N
    TEMP1=TEMP1-U(J)
    C1=X(J)
    SUMX=SUMX/(N-1)
    XX=SUMX/(N-1)
    X(J)=0.
    TEMP4=0.
DO 106 K=1,N
    C2=((X(K)-XX)**2)
    IF(X(K).EQ.0.)C2=0.
    TEMP4=TEMP4+C2
CONTINUE
SSX=TEMP4/(N-2)
LLN=ALOG(TEMP5/(N-1))
PLN(J)=(LLN*N)-(LLN*PLN(J))
SUM=SUM+PLN(J)
TEMP1=A
X(J)=C1
CONTINUE
PLNBAR=SUM/N
SUM=0.
DO 107 J=1,N
    SUM=SUM+(PLN(J)-PLNBAR)**2

```


107

```

CONTINUE
S2LN=SUM(N-1)
AJEB=(EXP(PLNBAR)/(1.+EXP(PLNBAR)))
GO TO 109
TEMP7=TEMP7+AJEB

```

C C COMPUTE CONFIDENCE LIMITS FOR JACKKNIFE PROCEDURE

```

CALL CONF(PSBAR,S2LN,ALPHA,CLS,CUS)
ALS=(EXP(CLS)/(1.+EXP(CLS)))
AUS=(EXP(CUS)/(1.+EXP(CUS)))
DELTAI(I)=AUS-ALS
SCORE1=0.
IF(AUS.GE.CNF.AND.ALS.LE.CNF) SCORE1=1.

```

C C COMPUTE CONFIDENCE LIMITS FOR LNlj PROCEDURE

```

CALL CONF(PLNBAR,S2LN,N,ALPHA,CLLN,CULN)
ALLN=(EXP(CLLN)/(1.+EXP(CLLN)))
AULN=(EXP(CULN)/(1.+EXP(CULN)))
DELTAA2(I)=AULN-ALLN
SCORE2=0.
IF(AULN.GE.CNF.AND.ALLN.LE.CNF) SCORE2=1.

```

C C COMPUTE CONFIDENCE LIMITS FOR MOMENT PROCEDURE

```

CALL MOMENT(X,N,XBAR,XVAR,XM4)
CALL MOMENT(Y,N,YBAR,YVAR,YM4)
SESQ=((YVAR+XVAR)/N)+((1./((4.*N)))*(XM4+YM4)-(XVAR**2)-(YVAR**2))
*
PM=(YBAR+0.5*YVAR)-(XBAR+0.5*XVAR)
AJEC=EXP(PM)/(EXP(PM)+1.)
TEMP8=TEMP8+AJEC
XALPHA=(1.+CNF)/2.
CALL MDNRIS(XALPHA,YY,IER)
CL=P-M-Y*SQRT(SESQ)
CU=P-M+Y*SQRT(SESQ)
AL=EXP(CL)/(1.+EXP(CL))
AU=EXP(CU)/(1.+EXP(CU))
DELTAA3(I)=AU-AL
SCORE3=0.
IF(AU.GE.CNF.AND.AL.LE.CNF) SCORE3=1.
C
Q1=Q1+SCORE1
Q2=Q2+SCORE2
Q3=Q3+SCORE3
Q4=Q4+DELTAA1(I)
Q5=Q5+DELTAA2(I)
```


Q6=Q6+DELT A3(1)

100

CONTINUE COMPUTE AND PRINT FINAL STATISTICS

```

A JEA=TEMP6 /RUN
A JEB=TEMP7 /RUN
A JEC=TEMP8 /RUN
CCOVER1=Q1 /RUN
COVER2=Q2 /RUN
COVER3=Q3 /RUN
AVG1=Q4 /RUN
AVG2=Q5 /RUN
AVG3=Q6 /RUN

```

```

Q3=0 i2 I=1 RUN
DC Q1=Q1+(DELTAI(I)-AVG1) ** 2
      Q2=Q2+(DELTAI(I)-AVG2) ** 2
      Q3=Q3+(DELTAI(I)-AVG3) ** 2

```

112

```

CONTINUE
VAR1=Q1/{RUN-1}
VAR2=Q2/{RUN-1}
VAR3=Q3/{RUN-1}
WRITE(6,7)
WRITE(6,7)
WRITE(6,7)
WRITE(6,7)
N,RUN

```

```

        WRITE(6,1)
        WRITE(6,2)
        WRITE(6,3)
        WRITE(6,2)
        FORMAT(3X,'N='12,4X,'RUN='14)
        FORMAT(1X,*)
        FORMAT(1X,*)
        * JACK KNIFE PROCEDURE STATISTICS RESULTS *
        *                                           *
        WRITE(6,4) COVER1
        WRITE(6,5) AVG1,VARI
        WRITE(6,6) AVG2,VAR2
        WRITE(6,7) A JEA
        WRITE(6,8)

```



```

      WRITE(6,8) AJEB
      WRITE(6,7)
      WRITE(6,7)
      WRITE(6,2)
      WRITE(6,2)
      WRITE(6,9)
      WRITE(6,4) COVER3
      WRITE(6,5) AVG3,VAR3
      WRITE(6,7)
      WRITE(6,8) AJEC
      IF(L.EQ.2) GO TO 199
      WRITE(6,11)
      GO TO 10
      4   FORMAT(0,3X,'COVERAGE IS ',F6.4)
      5   FORMAT(0,3X,'AVERAGE OF DELTA IS ',F6.4,' VARIANCE IS ',F6.4)
      6   FORMAT(1X,*LNJ PROCEDURE STATISTICS RESULTS *0,)
      7   FORMAT(*)
      8   FORMAT(3X,*POINT AVAILABILITY STATISTICS RESULTS *0,)
      9   FORMAT(1X,*MOMENT PROCEDURE STATISTICS RESULTS *0,)
      11  FORMAT(*1)
      STOP
      END

C**** SUBROUTINE FOR CONFIDENCE LIMITS OF PSEUDOVALUE PARAMETERS ****
C**** SUBROUTINE COMPUTES THE FOURTH MOMENT OF A DATA VECTOR. ***
C**** SUBROUTINE COMPUTES THE FOURTH MOMENT OF A DATA VECTOR. ***
C**** SUBROUTINE COMPUTES THE FOURTH MOMENT OF A DATA VECTOR. ***

      SUBROUTINE CONF(ZBAR,ZVAR,N,ALPHA,LB,UB)
      INTEGER N,IER
      REAL ZBAR,ZVAR,ALPHA,B,T,LB,UB
      B=FLOAT(N-1)
      CALL MDST1(ALPHA,B,T,IER)
      LB=ZBAR-(T*SQRT((ZVAR/N)))
      UB=ZBAR+(T*SQRT((ZVAR/N)))
      RETURN
      END

CONTINUE
      M4=SUM2*((AN-2)*AN+3)/(AN-1)*(AN-2)*(AN-3)/(2*(AN-3)*(AN-2)*(AN-3))
      M4=M4-VAR*VAR*3.**((AN-2)*AN+3)/(AN-1)*(AN-2)*(AN-3)/(2*(AN-3)*(AN-2)*(AN-3))

```


199 RETURN

END

C***** SUBROUTINE LNLJ(IS1, IS2, N, XMU, XSD, U, H, U, Y, D, X, UBAR, YBAR, DBAR,
* XVAR, XBAR, YVAR, XVAR, XBAR, YBAR, DBAR, YBAR, DBAR,
* SUM1, SUM2, SUM3, SUM4, IS1, IS2)
* DIMENSION U(25), D(25), W(25), W1(25), XX(25), X(25), Y(25)
INTEGER IS1, IS2, N, IS1, IS2
REAL U, D, X, UBAR, DBAR, XBAR, S2X, SUM1, SUM2, SUM3, SUM4, XMU, XVAR, H,
* XSD, YVAR, SUM5, SUM6

* SUM1=0.

SUM2=0.

SUM3=0.

SUM4=0.

DO 100 I=1,N

CALL LEXPN(IS1, Z, 1, 1, 0)

U(I)=Z*XUM

Y(I)=ALOG(U(I))

CALL LNORM(IS2, ZZ, 1, 1, 0)

XX(I)=XMU+(ZZ*XSD)

W(I)=EXP(XX(I))

W1(I)=W(I)*(1+H*W(I))

D(I)=3.844231*W1(I)/(3.844231+H*29.556224)

X(I)=ALOG(D(I))

SUM1=SUM1+U(I)

SUM2=SUM2+D(I)

SUM3=SUM3+Y(I)

SUM4=SUM4+X(I)

SUM5=SUM5+U(I)

UBAR=SUM1/N

DBAR=SUM2/N

YBAR=SUM3/N

XBAR=SUM4/N

SUM5=0.

SUM6=0.

DO 110 I=1,N

SUM5=SUM5+(X(I)-XBAR)**2

SUM6=SUM6+(Y(I)-YBAR)**2

CONTINUE

XVAR=SUM5/(N-1)

YVAR=SUM6/(N-1)

IS1=IS1

IS2=IS2

RETURN

END


```

*****
THIS SUBROUTINE COMPUTES THE BIWEIGHT ESTIMATE OF THE GIVEN
DATA VECTOR, AND USING THIS ESTIMATE COMPUTES THE CONFIDENCE
LIMITS.
*****  

ARGUMENTS:  

X   ; INPUT DATA VECTOR (SIZE N)  

C   ; CONSTANT DIVISOR (6 OR 9)  

C   ; TYPE I ERROR FOR T TEST  

C   ; BIWEIGHT ESTIMATE  

XHAT  

C   ; POPULATION VARIANCE ESTIMATE  

C   ; VARIANCE OF BIWEIGHT ESTIMATE XHAT  

S2BI  

C   ; LOWER BOUND ON BIWEIGHT ESTIMATE XHAT  

LB  

C   ; UPPER " "  

UB  

C   ; MEAN ABSOLUTE DEVIATION  

MAD  

U   ; NORMALIZED VALUES  

C   ; WEIGHTS  

W   ; CONSTANT FOR STOPPING RULE  

EPS  

C   ; SUBROUTINE BIWGT(X,N,C,ALPHA,XHAT,S2BI,LB,UB)  

C   ; INTEGER N, I IER  

C   ; DIMENSION X(25),X1(25),U(25),W(25),Y(25)  

REAL X,S,C,XHAT,XHAT1,SUM1,W,U,DIF,LB,UB,T,x1,S2BI,EPS,MAD,  

* EPS=0.0001  

CALL MEDIAN(X,N,XHAT1)  

J=1  

DC 10  I=1 N  

      Y1(I)=ABS(X(I)-XHAT1)  

10  CONTINUE  

CALL MEDIAN(Y1,N,S)  

SUM=0  

SUM1=0  

DO 15 I=1,N  

      U(I)=(X(I)-XHAT1)/(C*S)  

      IF(ABS(U(I)) GT 1.0) U(I)=1.  

      W(I)=(1.0-(U(I)*2))/2  

      SUM=SUM+W(I)  

      SUM1=SUM1+W(I)*X(I)  

15  CONTINUE  

XHAT=SUM1/SUM  

DIF=XHAT-XHAT1  

IF(ABS(DIF) LE EPS) GO TO 30  

IF(J GT 35) GO TO 30  

XHAT1=XHAT  

J=J+1  

GO TO 1

```


30

```

CONTINUE
DO 40 I=1,N
  X1(I)=X(I)-XHAT
  Y(I)=ABS(X1(I))
CONTINUE
CALL MEDIAN(Y,N,MAD)
DO 45 I=1,N
  U(I)=X(I)/(C*MAD)
  IF(ABS(U(I)).GE.1.)U(I)=1.
CONTINUE
SUM=0.

```

```

SUM=0.
DO 50 I=1,N
  IF(U(I) EQ 1.0) GO TO 47
  TOT=(X1(I)*I*2)*(1.-U(I)*I*2)*(1.-U(I)*I*4)
  TOT=-(X1(I)*I*2)*(1.-U(I)*I*2)*(1.-U(I)*I*4)
  GO TO 48
CONTINUE
46  TOT=0.
TOT=0.
TOT=0.
SUM=SUM+TOT
SUM1=SUM1+TOT

```

48

```

CONTINUE
S2BI=SUM/(SUM1*(SUM1-1.))
S2BI=S2BI*N
DF=0.7*(N-1)
CALL MDST(I,ALPHA,DF,IER)
LB=XHAT-(I*SQRT(S2BI))
UB=XHAT+(I*SQRT(S2BI))
RETURN
END

```

```

*****
THIS SUBROUTINE COMPUTES THE MEDIAN OF A GIVEN DATA VECTOR.
*****
SUBROUTINE MEDIAN(X,N,MED)
INTEGER N,IQ1,M1
REAL X(N),MFD
CALL PXSORT(X,1,N)
IQ1=N/2
M1=1-MOD(N/2)
MED=(M1*X(IQ1)+X(IQ1+1))/(1+M1)
RETURN
END

```



```

C***** SUBROUTINE FOR WINSORIZING GIVEN DATA
C***** FOR WINSORIZING GIVEN DATA
C***** SUBROUTINE WINSOR (ZZ,N,G,AA)
C***** INTEGER N,G,MO,M1,M2,M3
C***** REAL ZZ(N),AA(N)
C***** CALL PX SORT(ZZ,1,N)
C***** MO=G+1
C***** DO 1 I=1,MO
C*****   AA(I)=ZZ(MO)
C***** CONTINUE
C***** M1=MO+1
C***** M2=N-G
C***** DO 2 I=M1,M2
C*****   AA(I)=ZZ(I)
C***** CONTINUE
C***** M3=M2+1
C***** DO 3 I=M3,N
C*****   AA(I)=ZZ(M2)
C***** CONTINUE
C***** RETURN
C***** END

```


APPENDIX B

LONG-TAILED LOG-NORMAL DISTRIBUTION

(1) Let W be log-normal random variable which

$\ln W \sim N(\mu, \sigma^2)$. k moment of W as follows,

$$E[W^k] = \exp(k\mu + 1/2 \cdot k^2 \sigma^2),$$

see [Ref. 9].

So,

$$E[W] = \exp(\mu + 1/2 \cdot \sigma^2)$$

$$E[W^2] = \exp(2\mu + 2\sigma^2)$$

(2) Create a stretched log-normal random variable

$$D = c \cdot W \cdot (1 + h \cdot W)$$

where c and h are constant and $0 < h < 1$.

$$E[D] = c \{E[W] + h \cdot E[W^2]\}$$

$$E[D] = c \{\exp(\mu + 1/2 \cdot \sigma^2) + h \cdot \exp(2\mu + 2\sigma^2)\}$$

$$c = E[D] \{ \exp(\mu + 1/2 \cdot \sigma^2) + h \cdot \exp(2\mu + 2\sigma^2) \}^{-1} \quad (a.1)$$

(3) On the other hand, we can write the median for which

is the function of the expected value as follows,

$$M = d(0.5) = \psi E[D]$$

$$M = c \cdot \exp(\mu) \cdot (1 + h \cdot \exp(\mu)) = \psi E[D] \quad (a.2)$$

where ψ is the median location factor of the new distribution. If we put equation (a.1) in equation (a.2), we obtain

$$\exp(\mu) \cdot (1 + h \cdot \exp(\mu))$$

$$\psi \cdot E[D] = \frac{E[D]}{\exp(\mu + 1/2 \cdot \sigma^2) + h \cdot \exp(2\mu + 2 \cdot \sigma^2)}$$

After the cancellations, we obtain

$$\psi \{ \exp(\mu + 1/2 \cdot \sigma^2) + h \cdot \exp(2\mu + 2 \cdot \sigma^2) \} = 1 + h \cdot \exp(\mu)$$

$$\psi \cdot \exp(1/2 \cdot \sigma^2) - 1 = h \cdot \exp(\mu) \cdot (1 - \psi \cdot \exp(2 \cdot \sigma^2))$$

$$1 - \psi \cdot \exp(1/2 \cdot \sigma^2)$$

$$h = \exp(-\mu) \frac{\psi \cdot \exp(2 \cdot \sigma^2) - 1}{\psi \cdot \exp(1/2 \cdot \sigma^2)}$$

$$\text{If } h=0, \text{ then } \psi = \exp(-1/2 \cdot \sigma^2)$$

$$1 + \exp(\mu)$$

$$\text{if } h=1, \text{ then } \psi = \frac{\exp(\mu + 2 \cdot \sigma^2) + \exp(1/2 \cdot \sigma^2)}{\exp(\mu + 2 \cdot \sigma^2) + \exp(1/2 \cdot \sigma^2)}$$

so, if we get

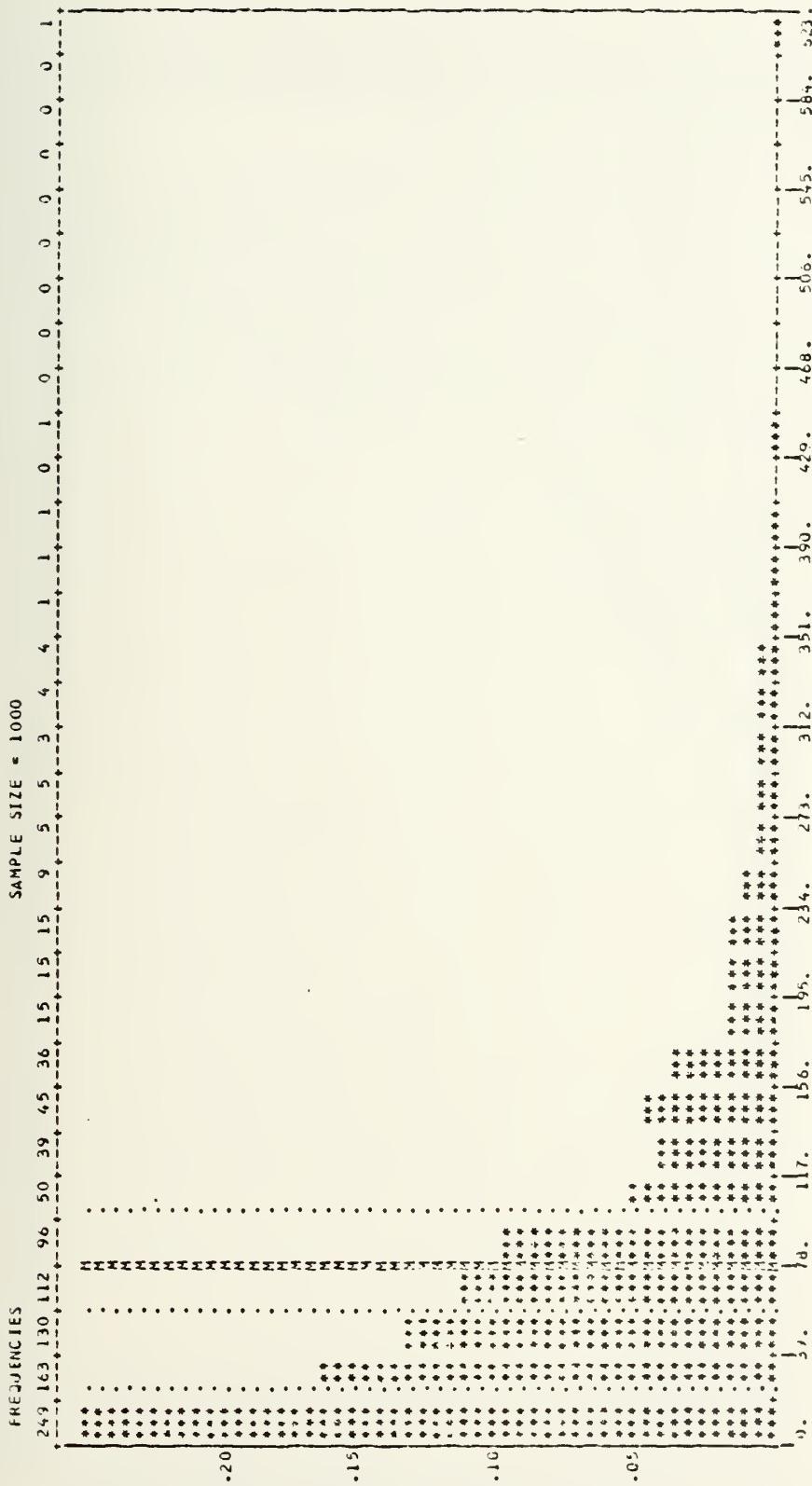
$$\exp(-1/2 \cdot \sigma^2) \leq \psi \leq \exp(\mu + 2 \cdot \sigma^2) + \exp(1/2 \cdot \sigma^2)$$

then we obtain h between 0 and 1.

APPENDIX C

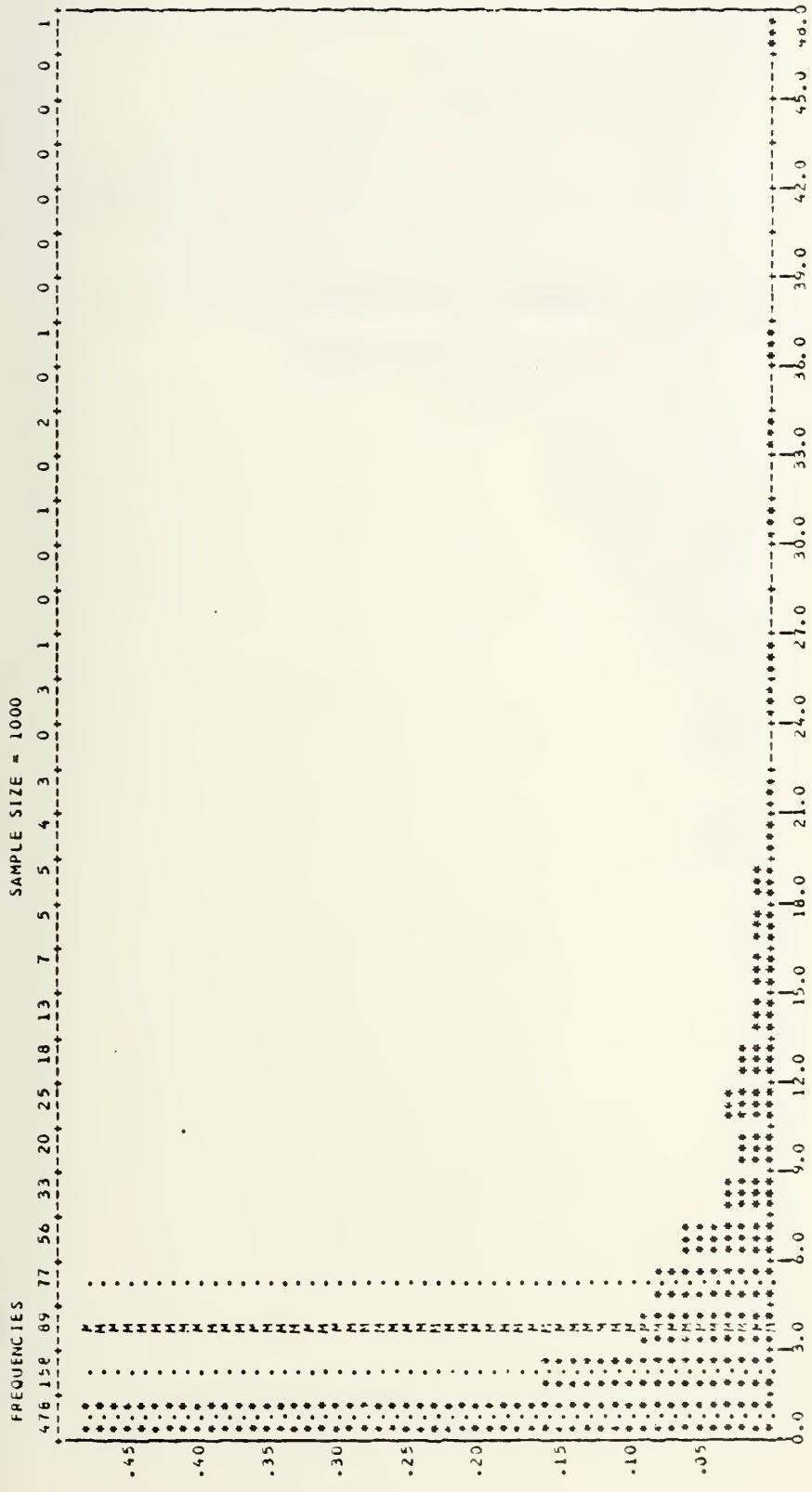
DISTRIBUTIONAL FIGURES

The LNLJ method fails for Exponential up and Gamma ($k=1/2$) down, Exponential up and Weibull ($k=1/2$) down, and Exponential up and Long-tailed Log-normal h down times cases. In order to demonstrate the distributional characteristics of these cases, histograms are obtained from simulation for up times, down times, log transformations of down times and pseudovalues. Also, histograms are repeated after grouping the observations in order to observe the grouping effect on the parameters. Distributional characteristics of these parameters such as means, variances, moments and shapes can be observed from these histograms.

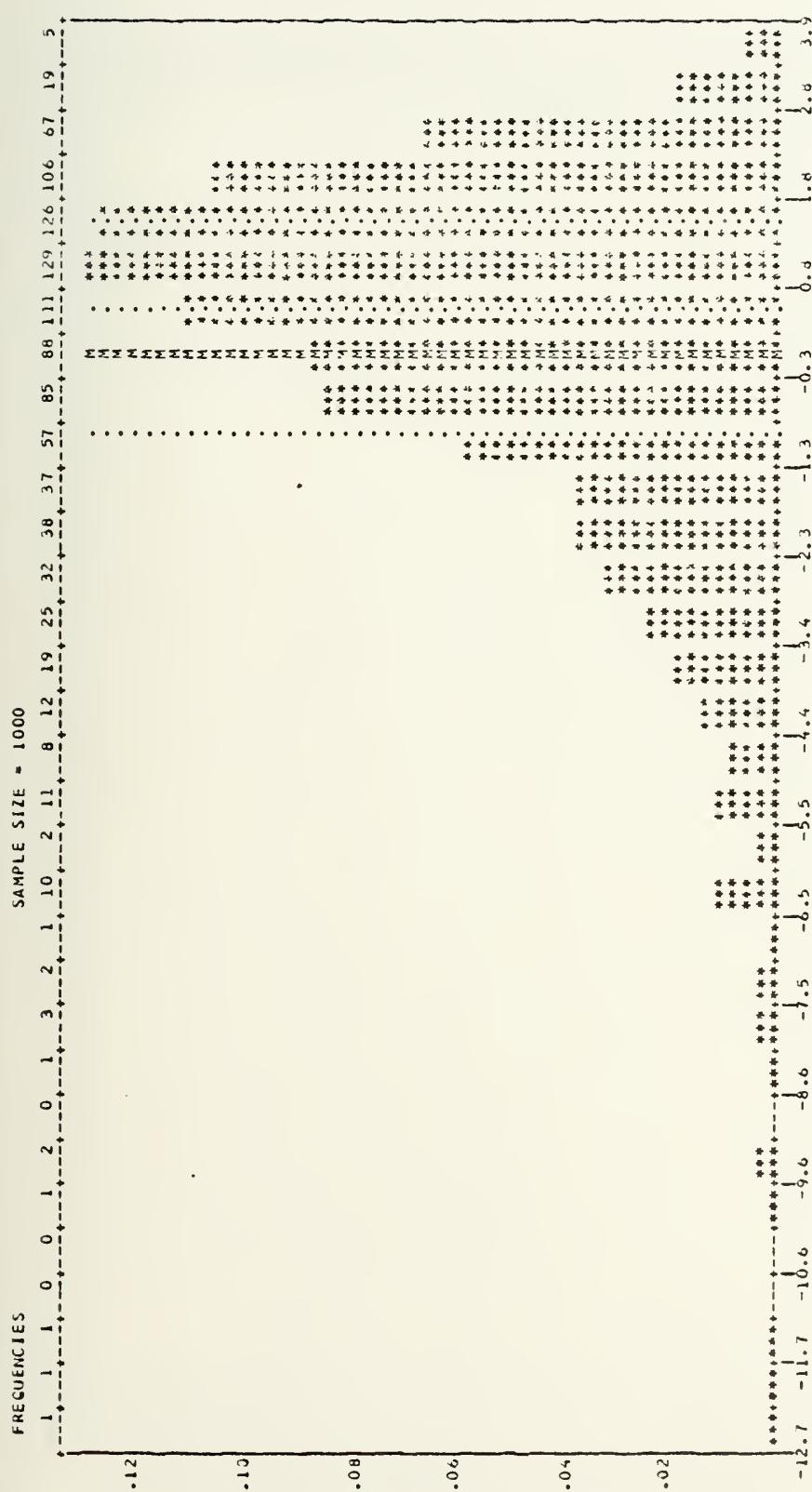


| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION |
|------------------|--------------|------------------------|----------------|----------------|-----------------------|
| | | VARIANCE | M ³ | M ⁴ | |
| MEAN | 7.200342E+01 | 5.014352E+01 | 6.931284E+05 | 6.60713E+02 | MINIMUM |
| MEDIAN | 5.25822E+01 | 7.081209E+01 | 2.315486E+08 | 6.61437E+02 | 10 QUANTILE |
| TRIMMED MEAN | 5.55718E+01 | 9.834518E+01 | 1.952012E+00 | 1.90377E+01 | 25 QUANTILE (HINGE) |
| MEAN DEV | 5.44753E+01 | 6.96774AE+01 | 6.225701E+00 | 5.78228E+01 | 50 QUANTILE (MEDIUM) |
| RANGE | 3.116370E+02 | 6.232620E+02 | 6.310265E+05 | 7.51728E+01 | 75 QUANTILE (HINGE) |
| GEM MEAN | 3.946955E+01 | 7.790559E+01 | 2.314912E+08 | 9.1792E+01 | 90 QUANTILE (MAXIMUM) |
| HARM MEAN | 9.710751E+00 | | | | |

1. Exponential Up times.

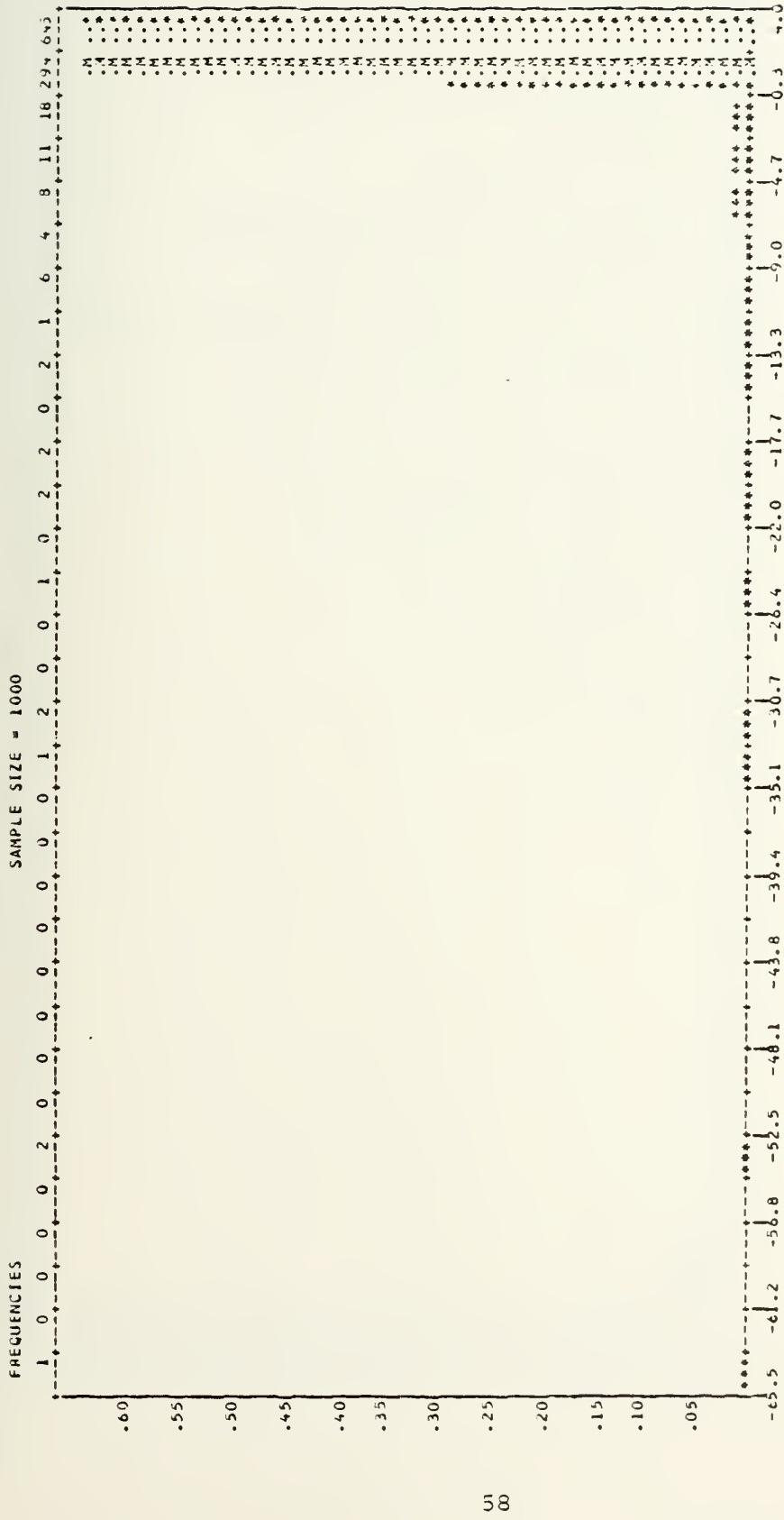


2. Gamma ($k=1/2$) down times.



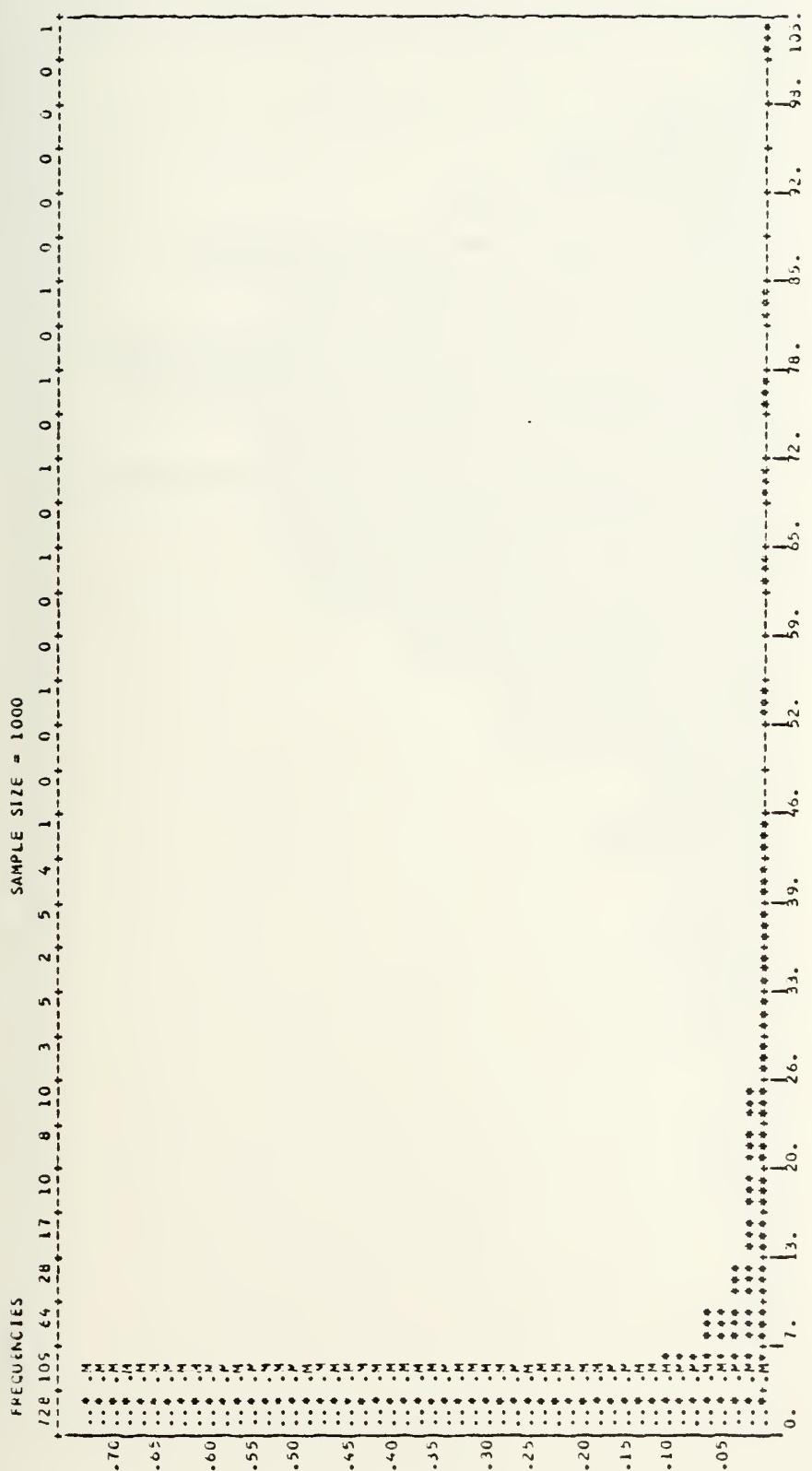
| CENTRAL TENDENCY | SPREAD | HIGHLY CENTRAL MOMENTS | | | DISTRIBUTION |
|------------------|---------------|------------------------|----------------|---------------|---------------------|
| | | M3 | M4 | SKEWNESS | |
| MEAN | 3.711124E-02 | 4.896603E+00 | -1.702157E+01 | -1.651129E+01 | MINIMUM |
| MEDIAN | 5.4531175E-01 | 2.128271E+00 | -1.651129E+01 | -1.571486E+00 | 10 QUANTILE |
| TRIMMED MEAN | 4.469476E-01 | 2.867343E+01 | -1.571486E+00 | -1.49135E+01 | 25 QUANTILE (HINGE) |
| HIGH MEAN | 4.743071E-01 | 1.602086E+00 | -3.886385E+00 | -5.43114E+01 | MEDIAN |
| MIDRANGE | -4.423813E+00 | 1.659143E+01 | -1.6297653E+01 | -7.50818E+01 | 75 QUANTILE (HINGE) |
| | | 2.518994E+00 | 1.645596E+02 | 2.08081E+02 | 90 QUANTILE |
| | | | | 3.871845E+00 | MAXIMUM |

3. Log-transforms of Gamma ($k = 1/2$) down times.



| CENTRAL TENDENCY | SPREAD | DISTRIBUTION | | | |
|---------------------|--------------------------|------------------------|----------------|----------------------------|-----------------|
| | | HIGHER CENTRAL MOMENTS | MINIMUM | QUANTILE | MAXIMUM |
| MEAN | 1.73652715×10^0 | VARIANCE | $2.195855E+01$ | $M_3 = -8.732340E-02$ | $-6.542455E+01$ |
| MEDIAN | $2.024046E+00$ | SD DEV | $2.000000E+00$ | $M_4 = 6.447670E+01$ | $2.976030E+01$ |
| TRIMMED MEAN | $2.643311E+00$ | COEF VAR | $2.608380E+00$ | $SKENNESS = -8.486306E+00$ | $1.453030E+01$ |
| MEAN DEVIATION | $2.011613E+00$ | MEAN DEV | $1.764929E+00$ | $KURTOSIS = 9.342186E+01$ | $2.770045E+00$ |
| RANGE | $3.000000E+00$ | RANGE | $6.952029E+00$ | $BETA1 = -7.020621E+01$ | $3.676145E+00$ |
| INTERQUARTILE RANGE | $3.000000E+00$ | MQSPREAD | $2.223145E+00$ | $BETA2 = 4.631066E+04$ | $3.982619E+00$ |

4. Pseudovalue of LNLIJ procedure for Exponential UP and Gamma ($k=1/2$) down times.



| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION |
|------------------|--------------|------------------------|----------------|----------------|--------------|
| | | M ₂ | M ₄ | M ₃ | |
| MEAN | 3.852026E+00 | 7.2541522E+01 | 3.346087E+03 | 2.68135E-15 | MINIMUM |
| MEDIAN | 0.752610E+01 | 8.56142E+00 | 2.33104E+03 | 2.26126E-15 | 10 QUANTILE |
| TRIMMED MEAN | 1.383789E+00 | 2.09613E+00 | 5.41680E+00 | 1.70073E-01 | 25 QUANTILE |
| MIDRANGE | 2.20324E+00 | 3.94235E+00 | 4.14153E+01 | 8.75291E-01 | {HINGE} |
| GEOMEAN | 5.231058E+01 | 1.46211E+02 | 3.33903E+03 | 3.606629E+00 | MEDIAN |
| HARM MEAN | 6.325018E-03 | 3.46481E+00 | 2.322034E+05 | 7.5 QUANTILE | {HINGE} |
| | | | | 9.619667E+00 | 90 QUANTILE |
| | | | | 1.046212E+02 | MAXIMUM |

5. Weibull ($k = 1/2$) down times.

FREQUENCIES

SAMPLE SIZE = 1000



CENTRAL TENDENCY

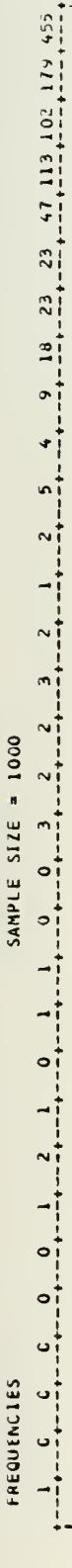
SPREAD

HIGHER CENTRAL MOMENTS

| | VARIANCE | STD DEV | COEF VAR | MEAN DEV | RANGE | MIDSPAN | DISTRIBUTION |
|----------|---------------|--------------|--------------|--------------|--------------|--------------|---------------|
| MEAN | -4.264925E-01 | 6.053996E+00 | 1.460987E+00 | 5.388985E+00 | 1.868863E+00 | 3.054797E+01 | -1.355162E+01 |
| MEDIAN | -1.286735E-01 | 2.460987E+00 | 5.388985E+00 | 1.868863E+00 | 1.51797E+01 | 3.054797E+00 | -1.483882E+02 |
| TRIMMEAN | -1.665249E-01 | 5.388985E+00 | 1.868863E+00 | 1.51797E+01 | 2.51092E+01 | 3.054797E+00 | -9.097620E+02 |
| MIDMEAN | -1.392045E-01 | 1.868863E+00 | 1.51797E+01 | 2.51092E+01 | 1.280142E+02 | 1.048695E+00 | -1.048695E+00 |
| MIDRANGE | -2.538138E+00 | 3.054797E+00 | 3.054797E+00 | 3.054797E+00 | 3.054797E+00 | 3.054797E+00 | -1.251092E+01 |

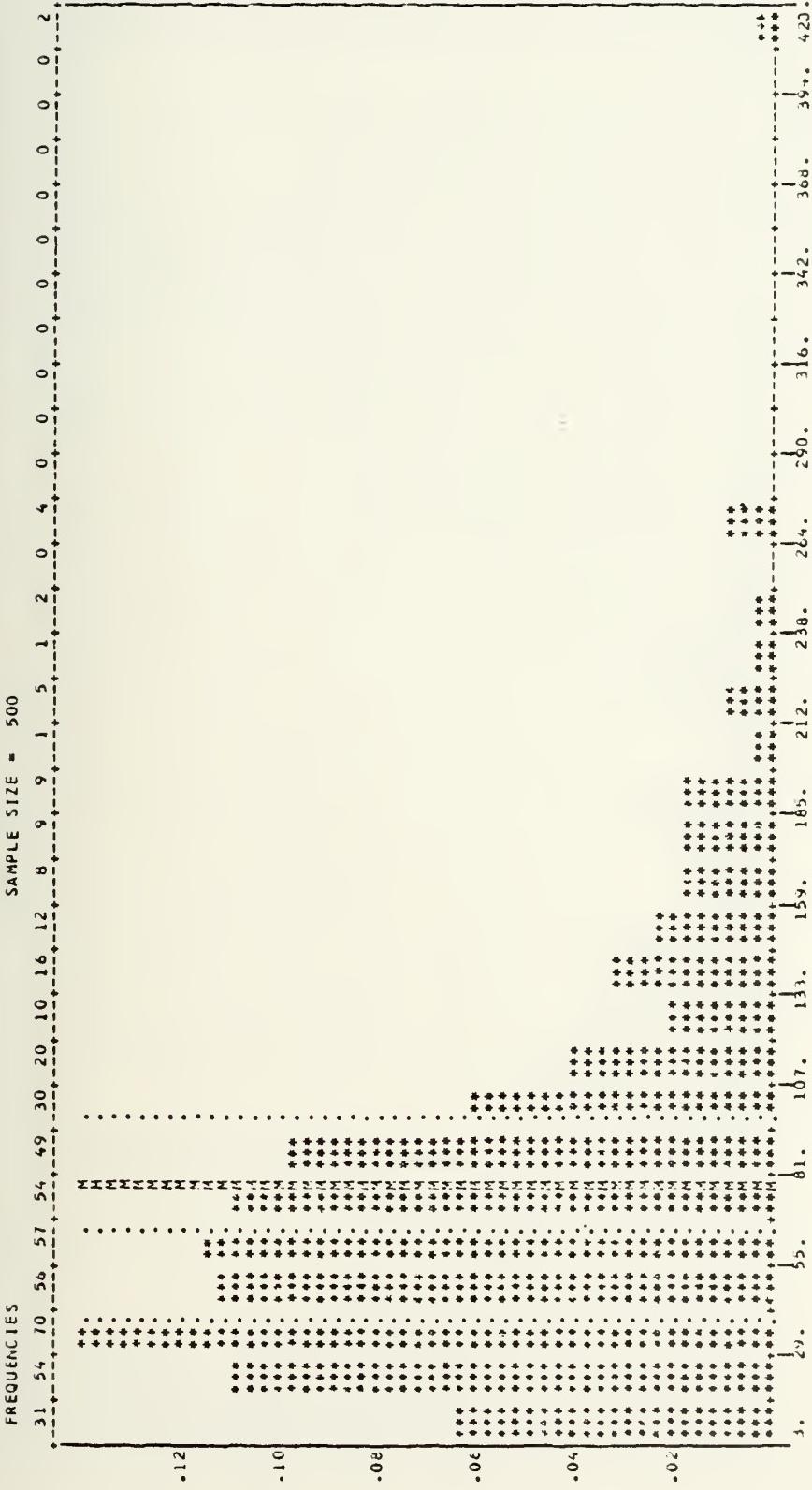
6. Log-transform of Weibull ($k = 1/2$) down times.

| | MINIMUM | 10 QUANTILE (HINGE) | 25 QUANTILE (HINGE) | 50 QUANTILE (MEDIAN) | 75 QUANTILE (HINGE) | 90 QUANTILE | MAXIMUM |
|--|--------------|---------------------|---------------------|----------------------|---------------------|--------------|--------------|
| | 7.105266E+01 | 7.381525E+00 | 7.177152E+00 | 7.128672E+00 | 7.128672E+00 | 7.128672E+00 | 4.650346E+00 |



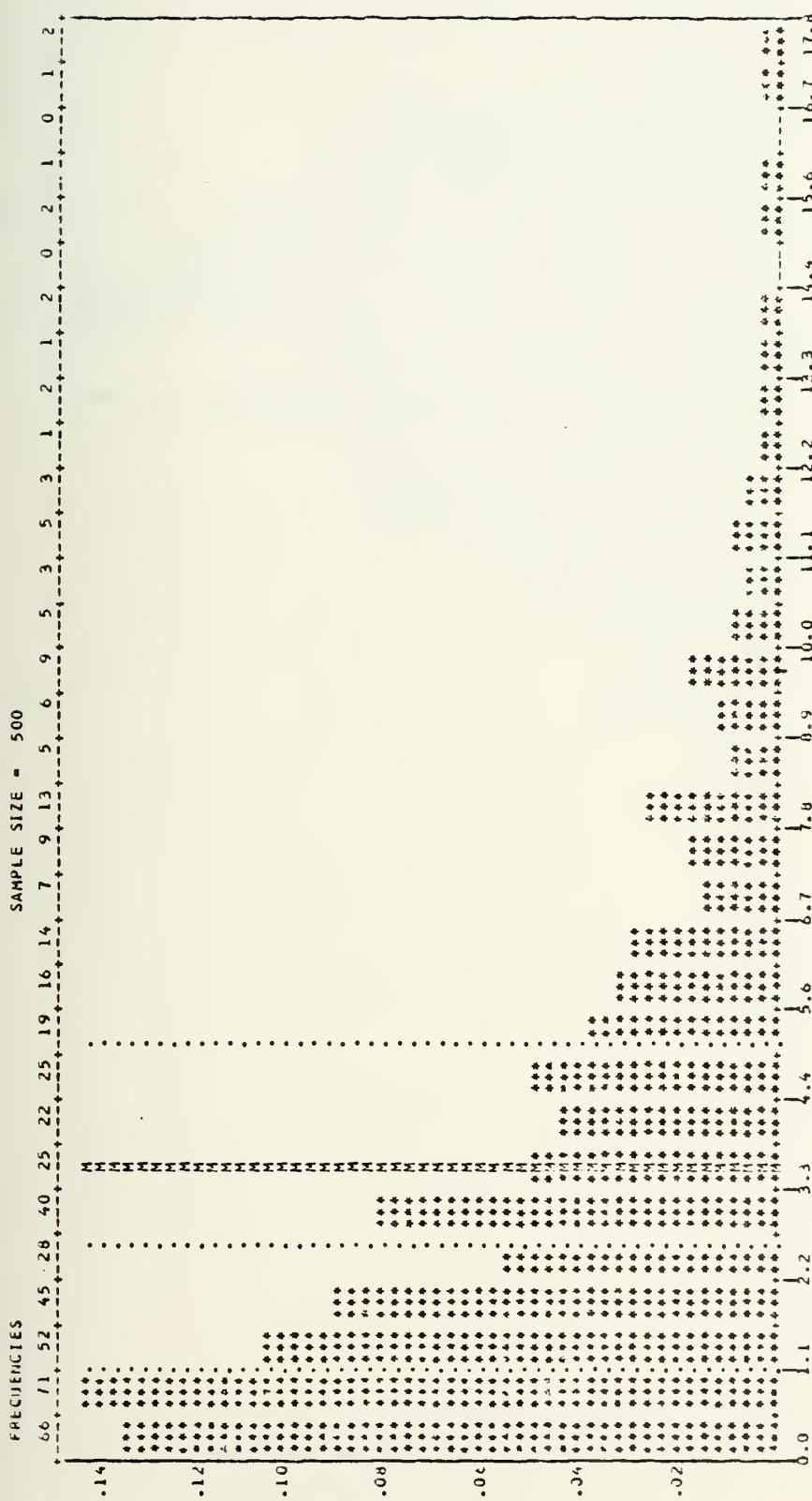
| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION |
|------------------|---------------|------------------------|---------------|---------------|---------------|
| | | M3 | M4 | MINIMUM | |
| MEAN | 1.677293E+00 | 1.74142E+01 | 7.767120E+02 | -2.761970E+02 | -3.698631E+01 |
| MEDIAN | 1.677293E+00 | 4.17302E+00 | 7.310970E+02 | -2.05232E+02 | -2.05232E+02 |
| TRIMMEAN | 2.692686E+00 | 2.45876E+00 | -3.80890E+02 | -2.52291E+01 | -2.52291E+01 |
| AVERAGE | 2.83621E+00 | 2.38257E+00 | 2.11511E+02 | -2.042270E+02 | -2.042270E+02 |
| MINRANGE | -1.619957E+01 | 4.15747E+01 | -2.75322E+02 | -4.37305E+02 | -4.37305E+02 |
| MIDSPREAD | 3.666016E+00 | 7.2486527E+02 | 7.2486527E+02 | -9.02349E+02 | -9.02349E+02 |

7. Pseudovalue of LNLJ procedure for Exponential up and Weibull ($k = 1/2$) down times.



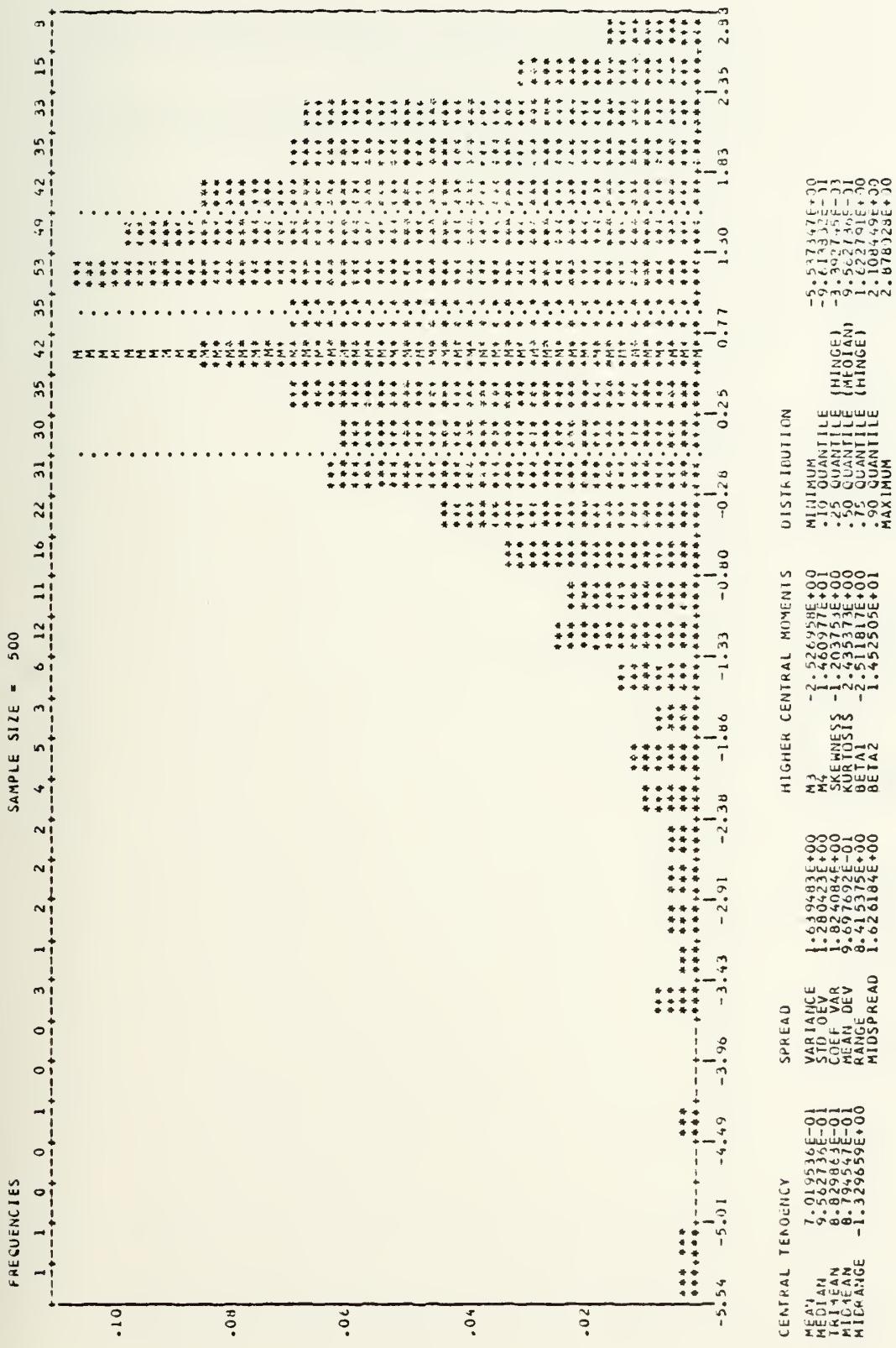
| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | DISTRIBUTION |
|------------------|--------------|------------------------|--------------|
| MEAN | 7.52109E+01 | 3.1371E+00 | MINIMUM |
| MEDIAN | 6.33212E+01 | 8.60129E+07 | QUANTILE |
| TRIMAN | 6.45268E+01 | 1.88200E+00 | QUANTILE |
| MIDSPAN | 6.37900E+01 | 6.0097E+00 | QUANTILE |
| MIDSPAN | 6.15084E+01 | 3.21285E+05 | QUANTILE |
| GEOM MEAN | 5.75424E+01 | 8.54390E+07 | QUANTILE |
| HARM MEAN | 3.946172E+01 | 4.20405E+02 | MAXIMUM |

8. Exponential up times after grouping.

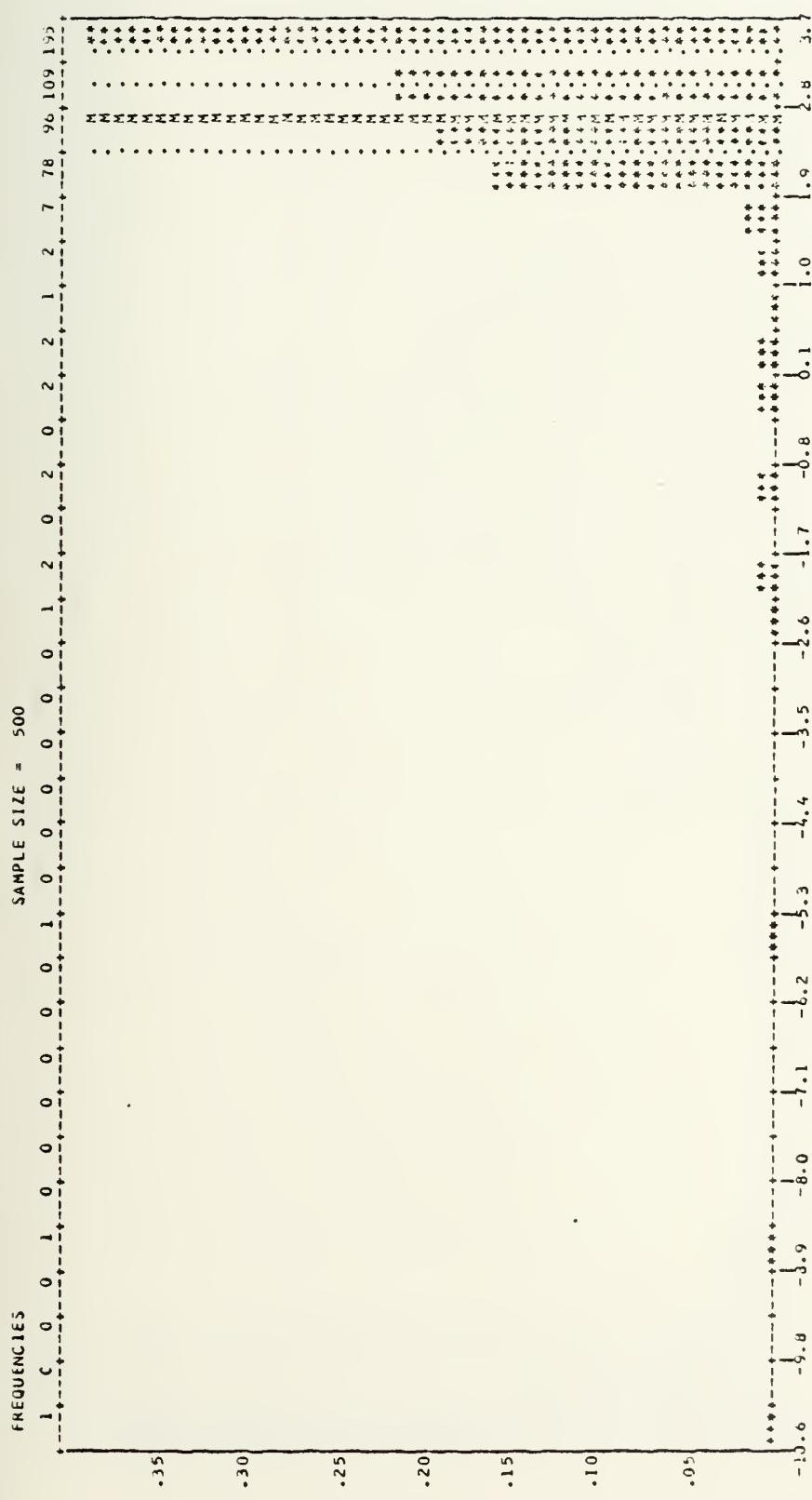


| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION |
|------------------|--------------|------------------------|--------------|--------------|----------------------|
| | | VARIANCE | KURTOSIS | BETA1 | |
| MEAN | 1.24117E+00 | 1.19337E+00 | 5.60055E+01 | 6.18526E+02 | MINIMUM |
| MEODIAN | 3.20121E+00 | 1.14665E+00 | 6.14956E+01 | 1.44560E+02 | 10 QUANTILE |
| TRIMMED MEAN | 2.97039E+00 | 9.445051E+00 | 1.49363E+00 | 2.82233E+00 | 25 QUANTILE (HINGE) |
| MEDIAN | 2.66529E+00 | 2.479712E+00 | 2.287337E+00 | 5.567337E+01 | 50 QUANTILE (MEDIUM) |
| HIGH RANGE | 3.89152E+00 | 4.77524E+00 | 5.580439E+01 | 7.50000E+00 | 75 QUANTILE (HINGE) |
| GEOMEAN | 2.01765E+00 | 4.070877E+00 | 6.580439E+02 | 8.23790E+01 | 90 QUANTILE |
| HARM MEAN | 4.318705E-01 | | | | MAXIMUM |

9. Gamma ($k = 1/2$) down times after grouping.

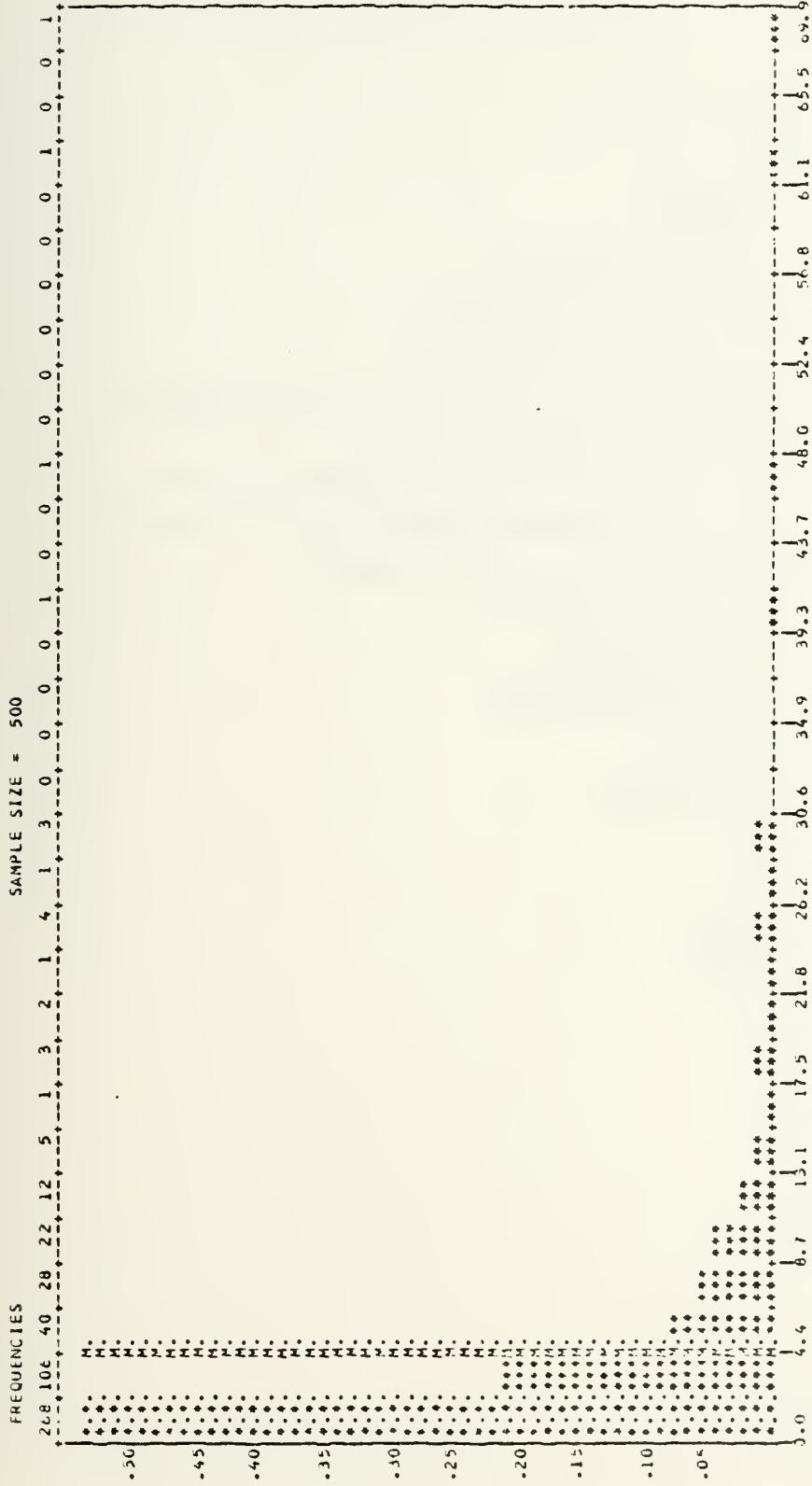


10. Log-transform of Gamma ($k = 1/2$) after grouping.



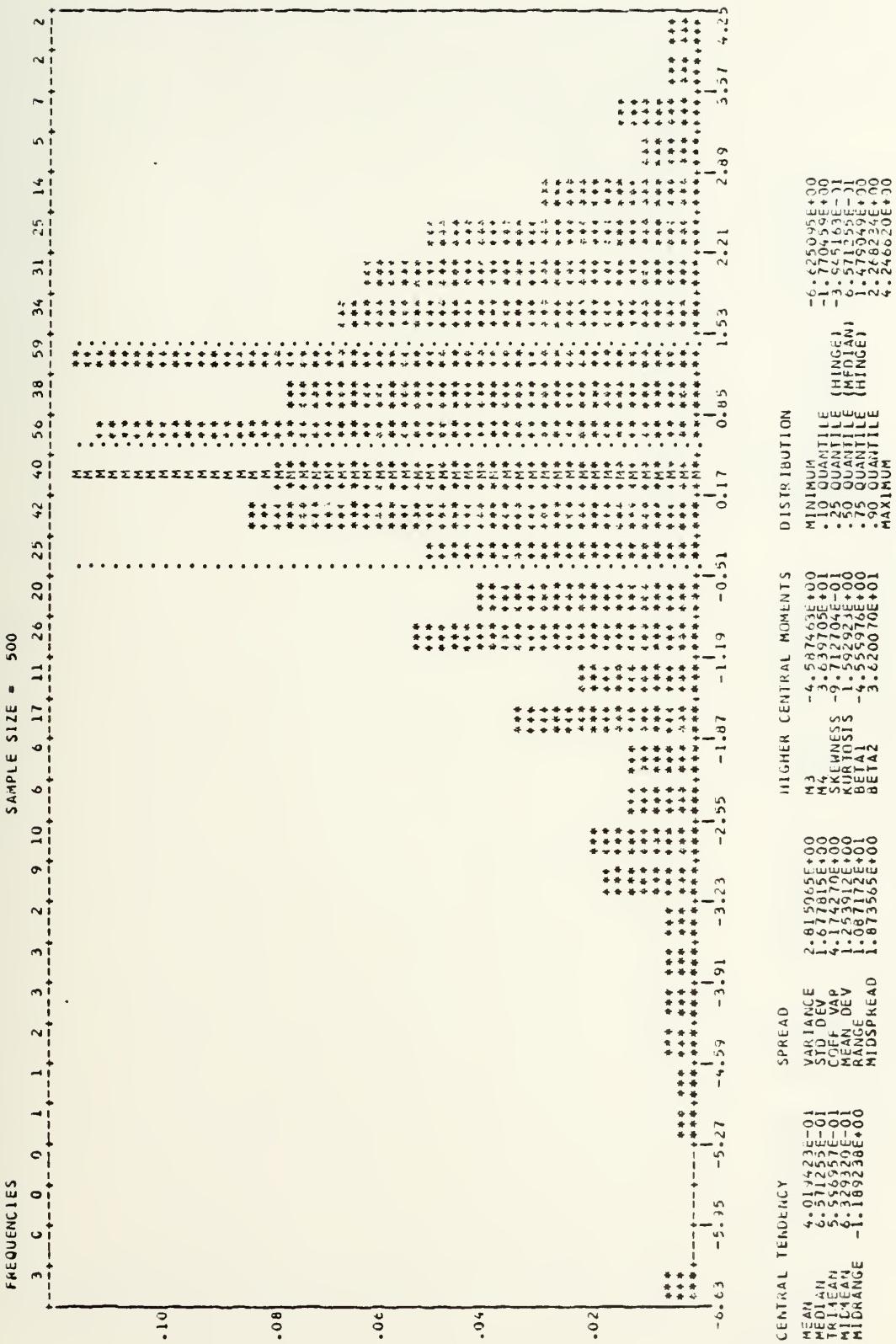
| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | | DISTRIBUTION |
|------------------|---------------|------------------------|----------|----------------|--------------|--------------|
| | | VARIANCE | STD DEV | COEF VAR | MEAN DEV | |
| MEAN | 7.751450E+00 | 1.322867E+00 | M3 | -1.012170E+01 | MINIMUM | -1.04991E+01 |
| MEDIAN | 2.700170E+00 | 1.152687E+00 | M4 | 1.63737E+02 | 0 QUANTILE | 2.13281E+00 |
| TRI4 FAN | 2.992783E+00 | 4.115019E+00 | SKENNESS | -6.6008752E+00 | .25 QUANTILE | 2.42481E+00 |
| MEAN | 3.002294E+00 | 5.853212E+01 | KURTOSIS | -6.2057862E+01 | .50 QUANTILE | 3.67471E+00 |
| MICRANGE | -3.446659E+00 | 5.1323446E+01 | BETA1 | -1.006106E+01 | .75 QUANTILE | 4.23074E+00 |
| MIDSPREAD | 9.982910E-01 | 1.1395660E+02 | BETA2 | 1.1395660E+02 | .90 QUANTILE | 3.53923E+00 |

- Pseudovalue of LNLJ procedure for Exponential Up and Gamma ($k = 1/2$) down times after grouping.

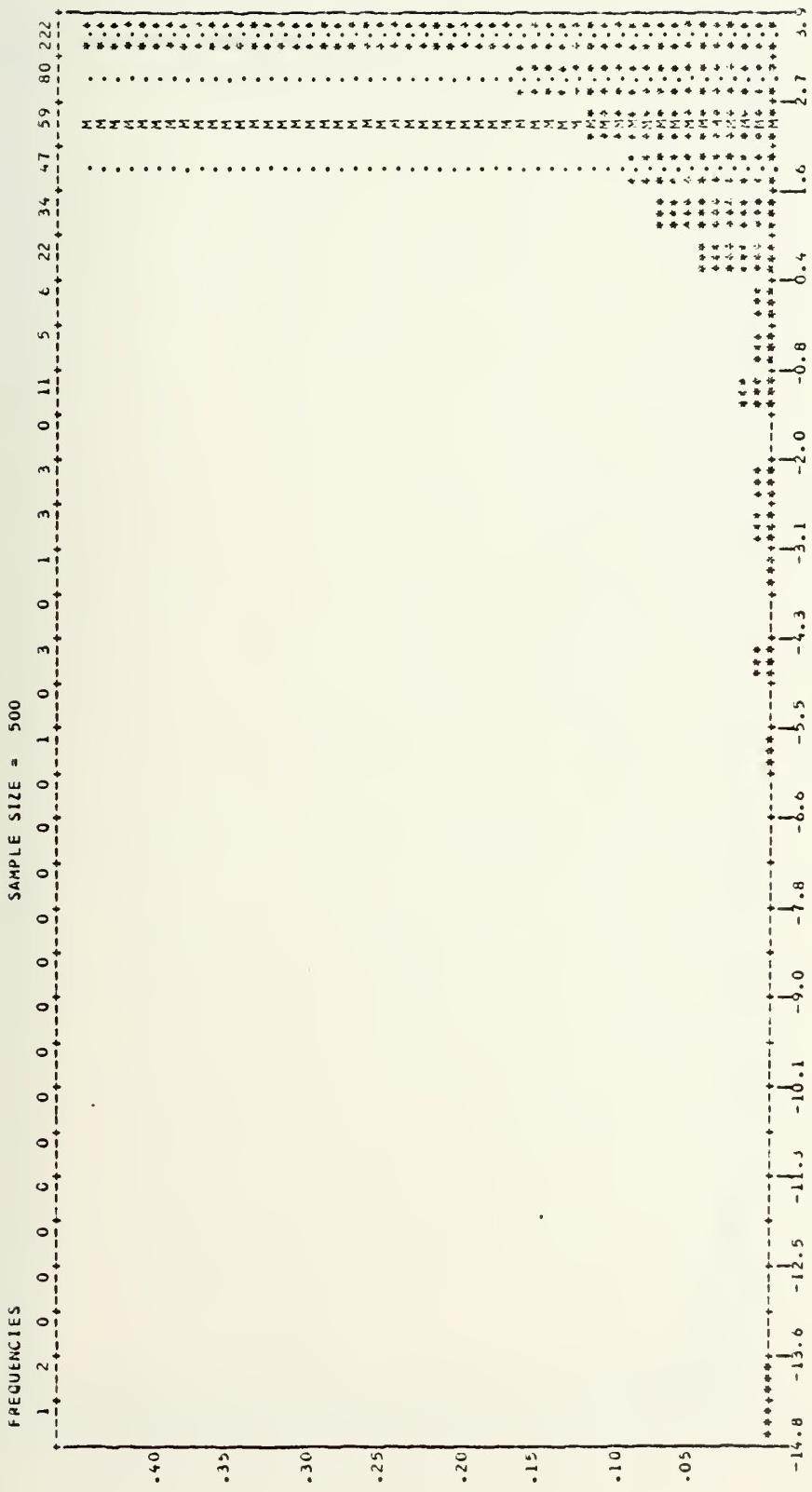


| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | | DISTRIBUTION |
|------------------|--------------|------------------------|--------------|-----------------------|----|--------------|
| | | M1 | M2 | M3 | M4 | |
| MEAN | 4.463084E+01 | 4.494118E+03 | 1.492118E+03 | 1.32695E-03 | | |
| MEDIAN | 4.329216E+00 | 6.680632E+00 | 5.01093E+00 | 1.0 QUANTILE | | |
| TRIMMED MEAN | 2.230384E+00 | 1.674289E+00 | 5.55557E+00 | .25 QUANTILE (HINGE) | | |
| STOKEZEN | 2.15354E+00 | 3.239119E+00 | 3.55557E+00 | .50 QUANTILE (MEDIAN) | | |
| MINIMUM | 3.493510E+01 | 6.986757E+01 | 1.481666E+02 | .75 QUANTILE (HINGE) | | |
| GCFM | 1.49625E+00 | 3.714598E+00 | 7.628187E+02 | .90 QUANTILE | | |
| HAFM MEAN | 1.32706E-01 | | | | | MAXIMUM |

12. Weibull ($k = 1/2$) down times after grouping.

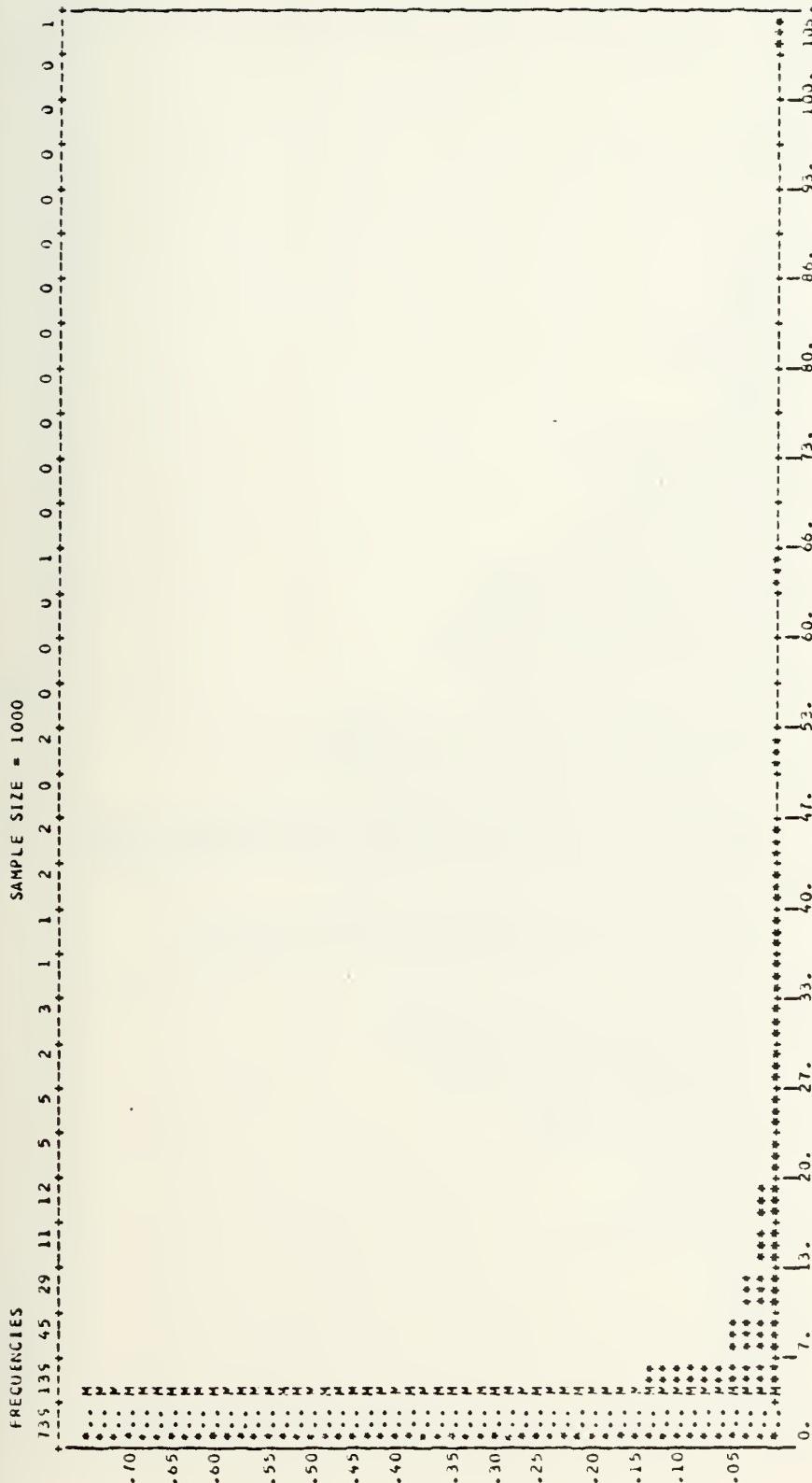


1.3. Log-transform of Weibull ($k = 1/2$) down times after grouping.



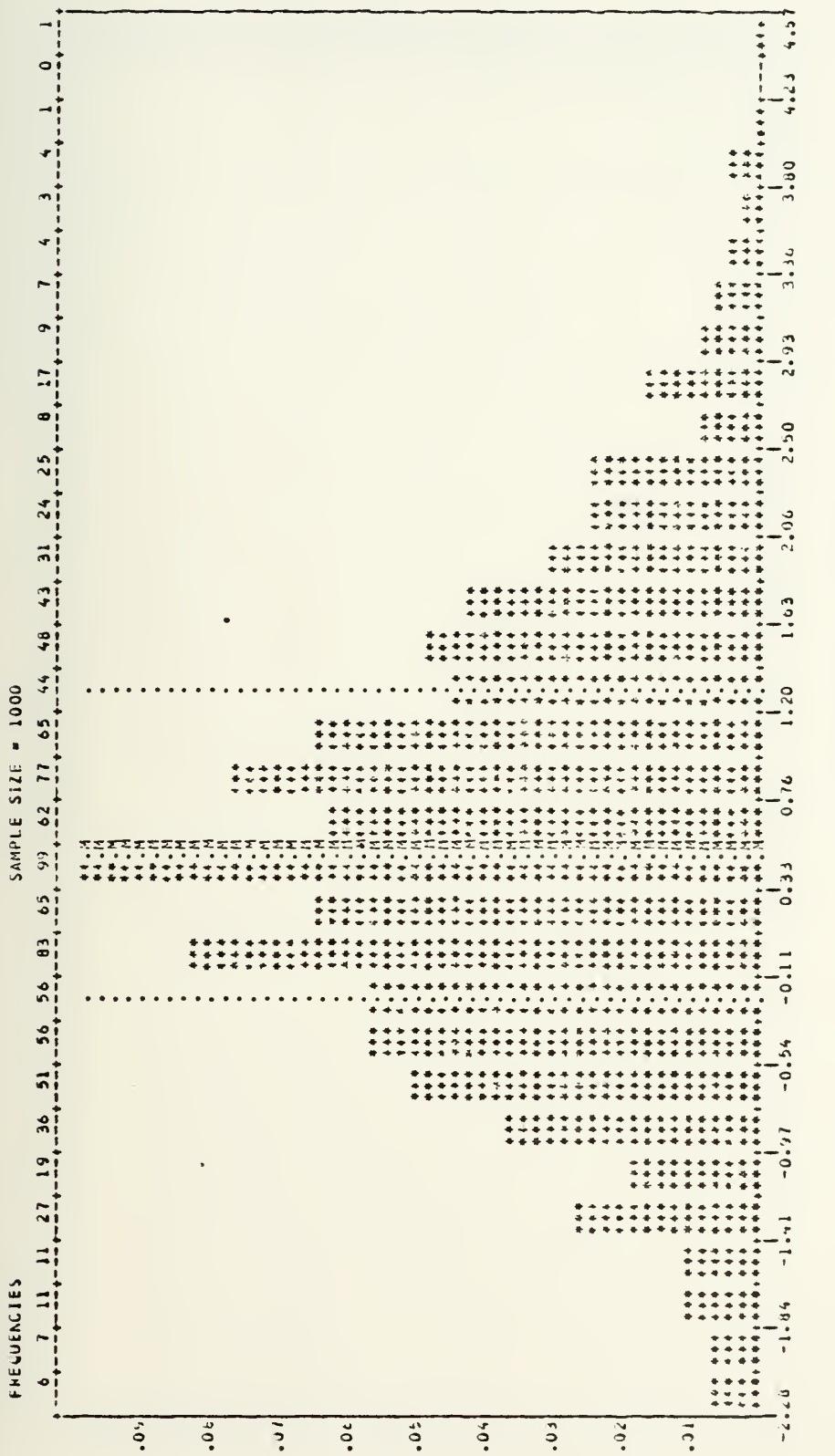
| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION |
|------------------|---------------|------------------------|----------------|----------------|---------------|
| | | M ₂ | M ₄ | M ₆ | |
| MEAN | 2.510654E+00 | 1.897115E+00 | -3.507424E+01 | -1.480640E+01 | |
| MEGAN | 3.09132E+00 | 1.97416E+00 | 5.281067E+01 | 8.391113E+01 | |
| TRMEAN | 2.940534E+00 | 1.862952E+00 | -4.561604E+00 | 1.196606E+01 | |
| MIDMEAN | 3.02356E+00 | 1.109790E+00 | 3.177208E+01 | 3.00332E+00 | |
| MIDRANGE | -5.456243E+00 | 1.869971E+01 | -3.488596E+01 | 3.68525E+00 | |
| | | MUSPREAD | 1.788816E+00 | 5.2440637E+02 | 3.8355939E+00 |
| | | | BETA2 | | 3.893311E+00 |
| | | | | | |

14. Pseudovalue of LNLJ procedure for Exponential up and Weibull ($k = 1/2$) down times after grouping.

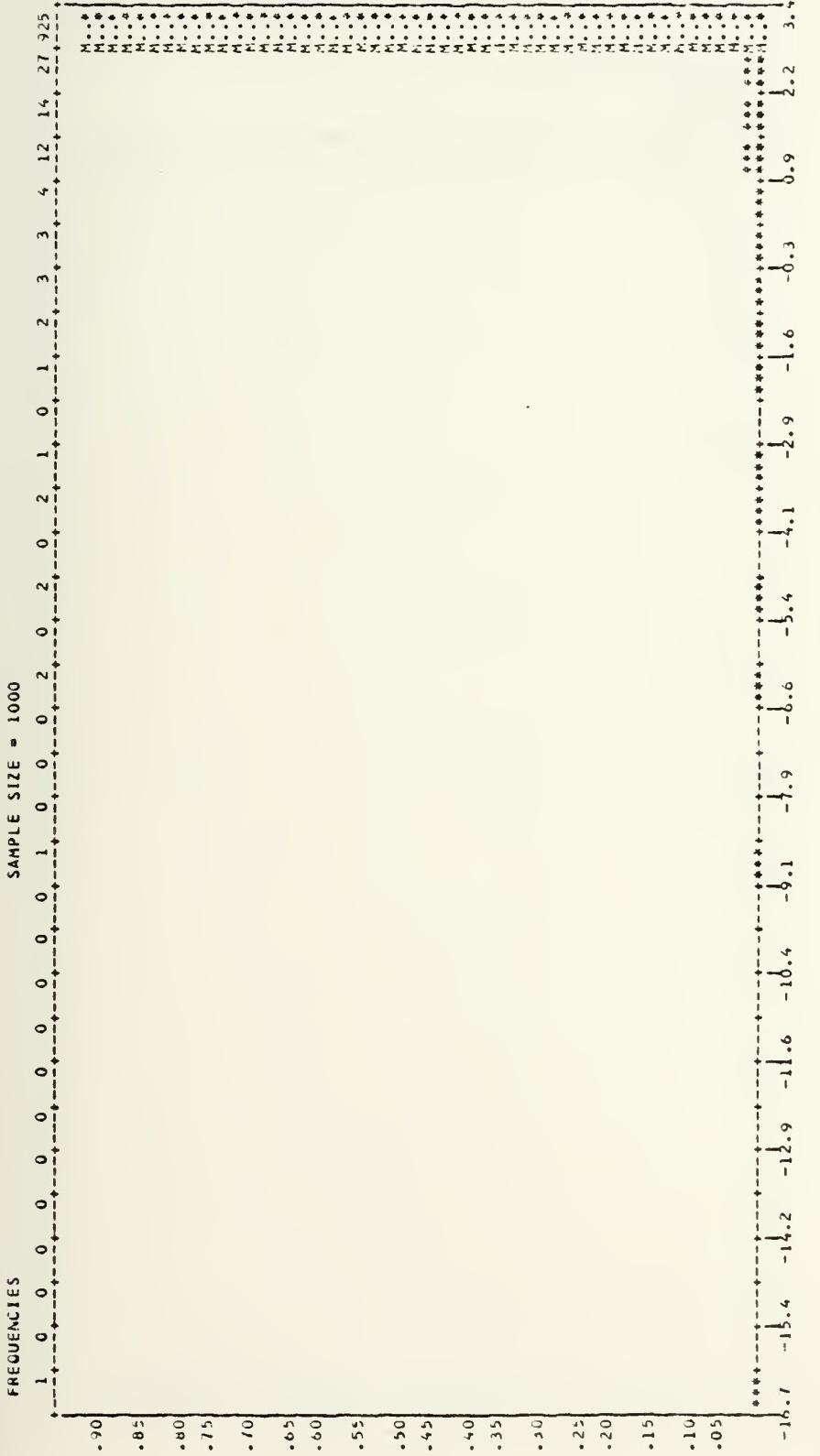


| CENTRAL TENDENCY | DISTRIBUTION | | | |
|------------------|----------------|------------------------|---------------|--------------|
| | SPEAO | HIGHER CENTRAL MOMENTS | VARIANCE | MINIMUM |
| MEAN | 3.651278E+000 | M3 | 2.11398E+03 | 1.042774E-01 |
| MEDIAN | 1.621004E+000 | M4 | 5.54466E+05 | 1.042774E-01 |
| STD DEV | 6.871669E+000 | SKEWNESS | 6.46165E+06 | 7.73942E-01 |
| CHEF VAR | 1.8711318E+000 | KURTOSIS | 6.79505E+01 | 1.042774E-01 |
| MEAN OEV | 2.881007E+000 | BETA1 | 1.062971E+01 | 1.042774E-01 |
| MEAN OEV | 1.201002E+000 | BETA2 | 2.152293E+05 | 1.042774E-01 |
| SD SPREAD | 5.320156E+001 | MEAN | 8.866526E+000 | 1.042774E-01 |
| HIGH TENDENCY | 7.752150E+001 | SD SPREAD | 9.645518E+001 | 1.042774E-01 |
| MEAN | 9.645518E+001 | MEAN | 9.645518E+001 | 1.042774E-01 |

15. Long-tailed Log-normal h down times.

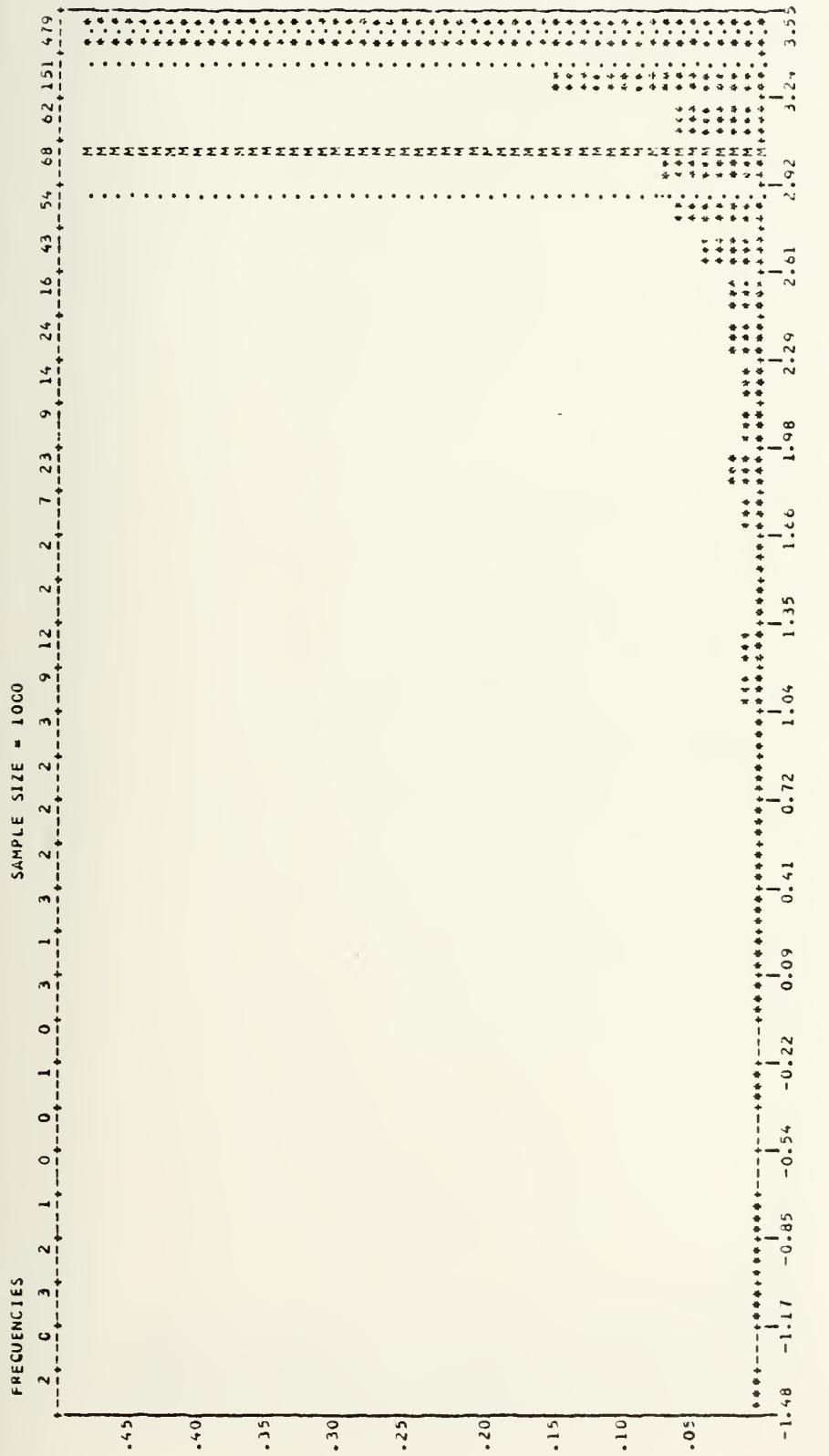


16. Log transform of long-tailed log-normal h down times.

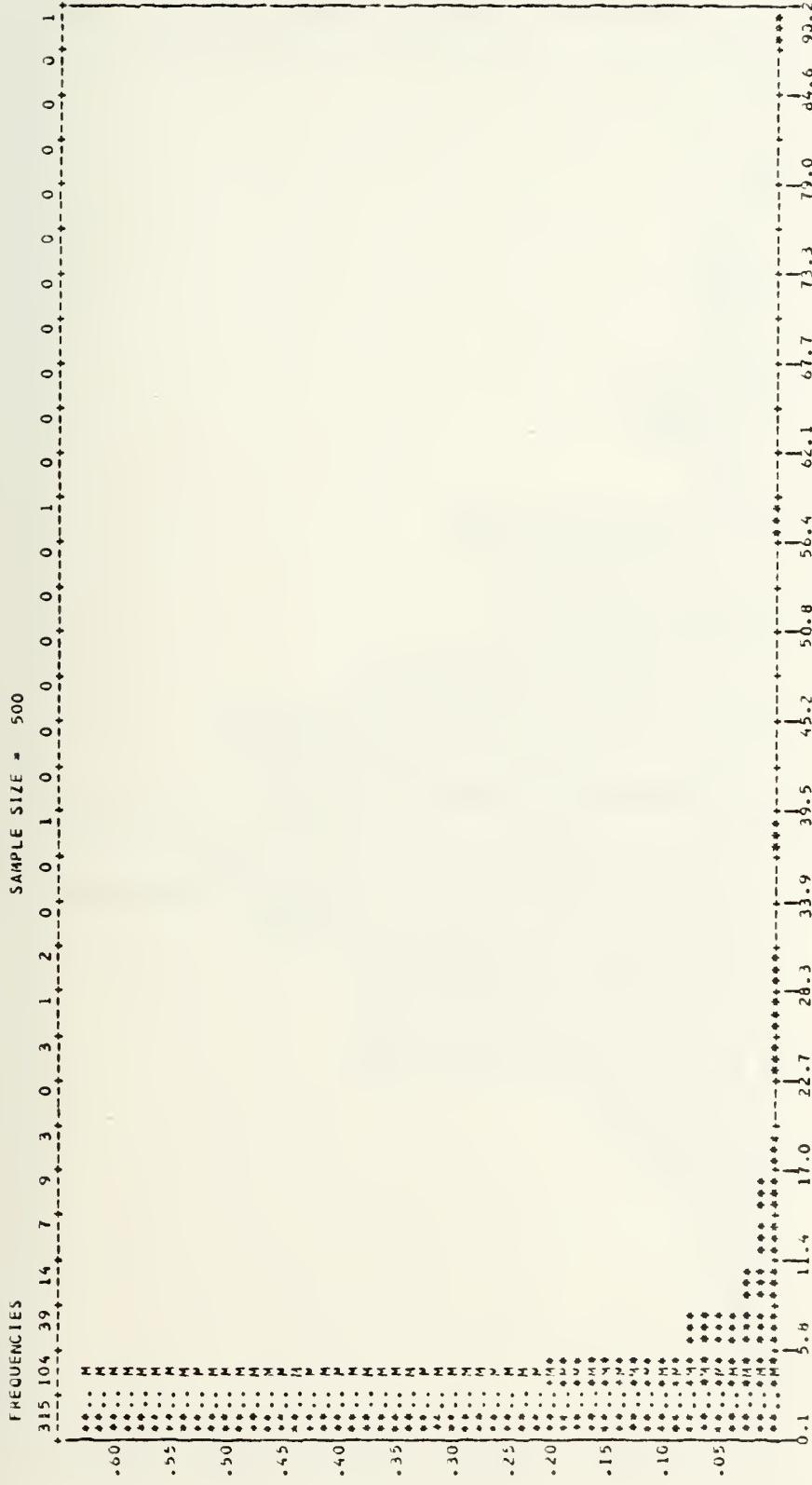


| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS DISTRIBUTION | | | |
|------------------|---------------|-------------------------------------|---------------|-----------------------|-------------------|
| | | M3 | M4 | MINIMUM | MAXIMUM |
| MEAN | 3.01315CE+00 | VARIANCE | 1.1267518E+00 | -1.66691CE+00 | -2.1-6.6691CE+00 |
| MEDIAN | 3.01315CE+00 | STD DEV | 1.1267518E+00 | .10 QUANTILE | .046875E+00 |
| TRIMMED MEAN | 3.0131712E+00 | COEF VAR | 3.624504E+00 | .25 QUANTILE (HINGE) | .3-0.624504E+00 |
| MEAN | 3.0131712E+00 | MEAN DEV | 3.1249611E+01 | .50 QUANTILE (MEDIAN) | .3-3.28124961E+00 |
| MEAN | 3.0131712E+00 | RANGE | 2.0699611E+01 | .75 QUANTILE (HINGE) | .3-2.0699611E+00 |
| MIDRANGE | -6.61938E+00 | MIDSPREAD | 2.814941E-01 | .90 QUANTILE | .3-6.342344E+00 |

17. Pseudovalues of JK procedure for Exponential up and long-tailed log-normal down times.

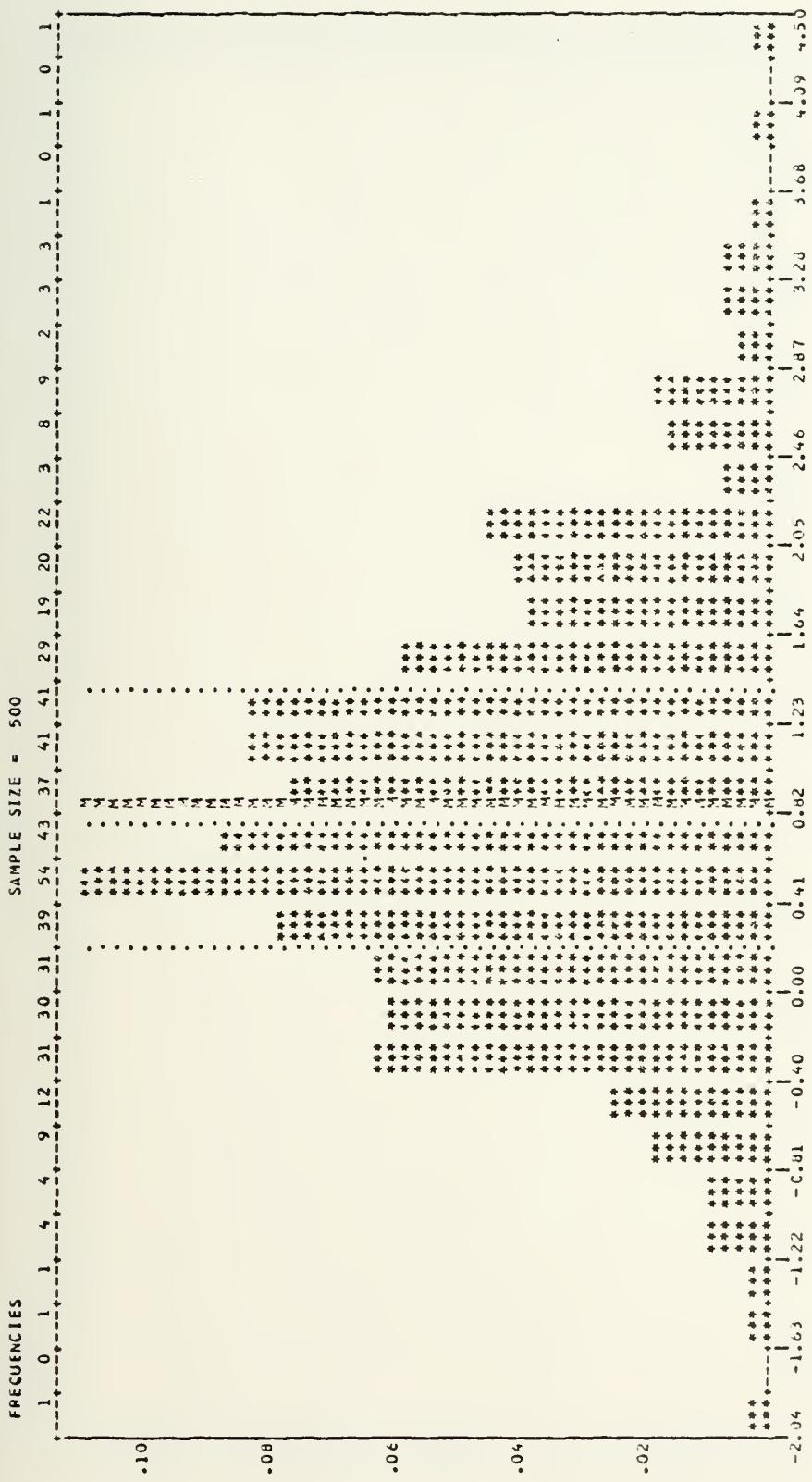


18. Pseudovalues for LNLJ procedure for Exponential up and long-tailed log normal down times.



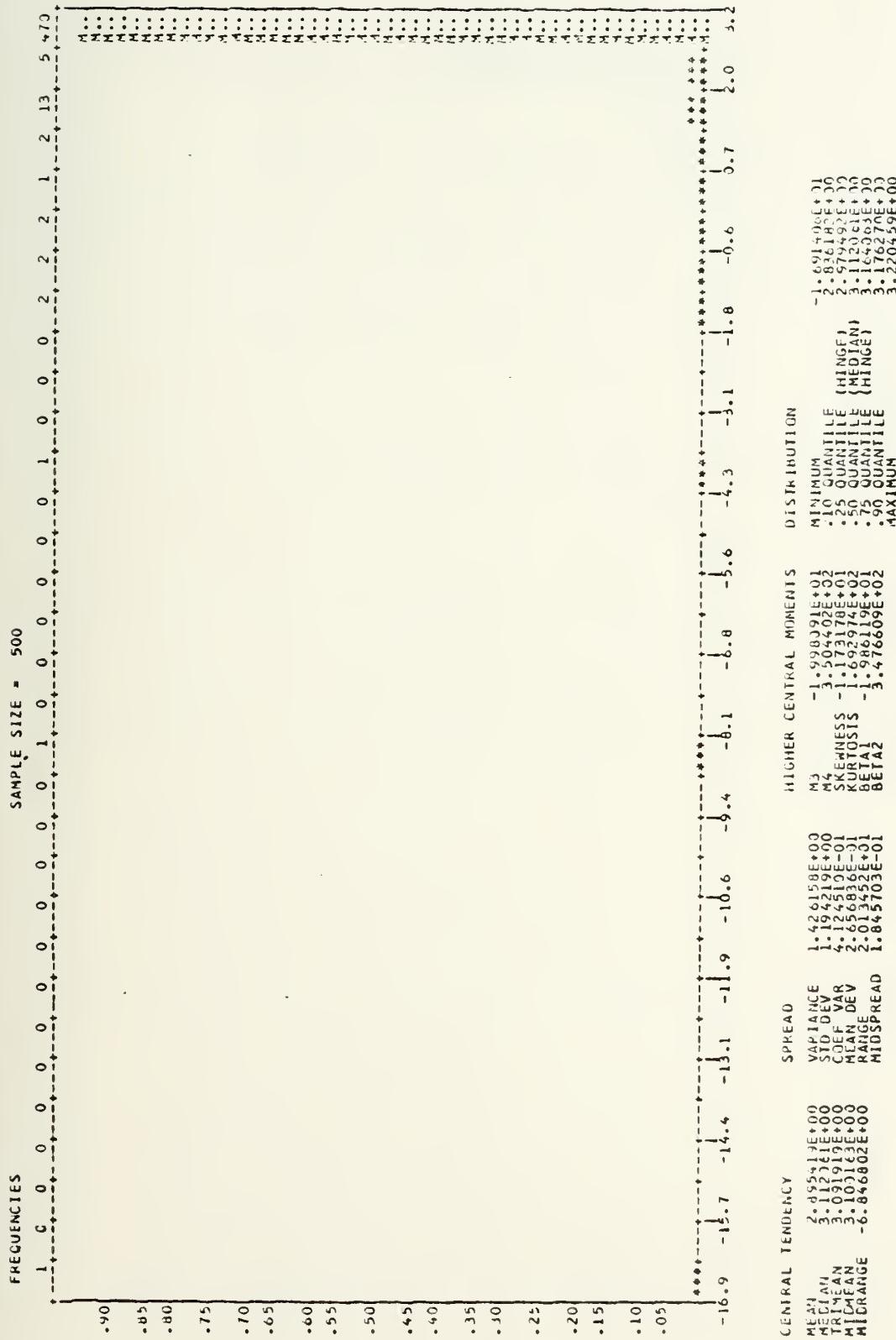
| CLINICAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION |
|-------------------|--------------|------------------------|--------------|--------------|--------------|
| | | M3 | M4 | MINIMUM | |
| MEAN | 3.97714E+00 | 3.99149E+01 | 1.88514E+03 | 1.30083E-01 | |
| MEDIAN | 2.15975E+00 | 6.31779E+00 | 1.32218E+03 | 1.47402E-01 | |
| TRIMMEAN | 2.36920E+00 | 6.62466E+00 | 1.47515E+03 | 1.236163E+00 | |
| MEAN | 2.34166E+00 | 6.63319E+00 | 8.18276E+01 | 2.15974E+00 | |
| MEAN | 2.34166E+00 | 6.63319E+00 | 1.83846E+03 | 3.92424E+00 | |
| MEAN | 2.34166E+00 | 6.63319E+00 | 1.340562E+05 | 7.503708E+00 | |
| MEAN | 2.34088E+00 | 6.63319E+00 | 1.340562E+05 | 9.022809E+01 | |
| HARM MEAN | 1.588791E+00 | | | | |

19. Long-tailed log-normal down times after grouping.

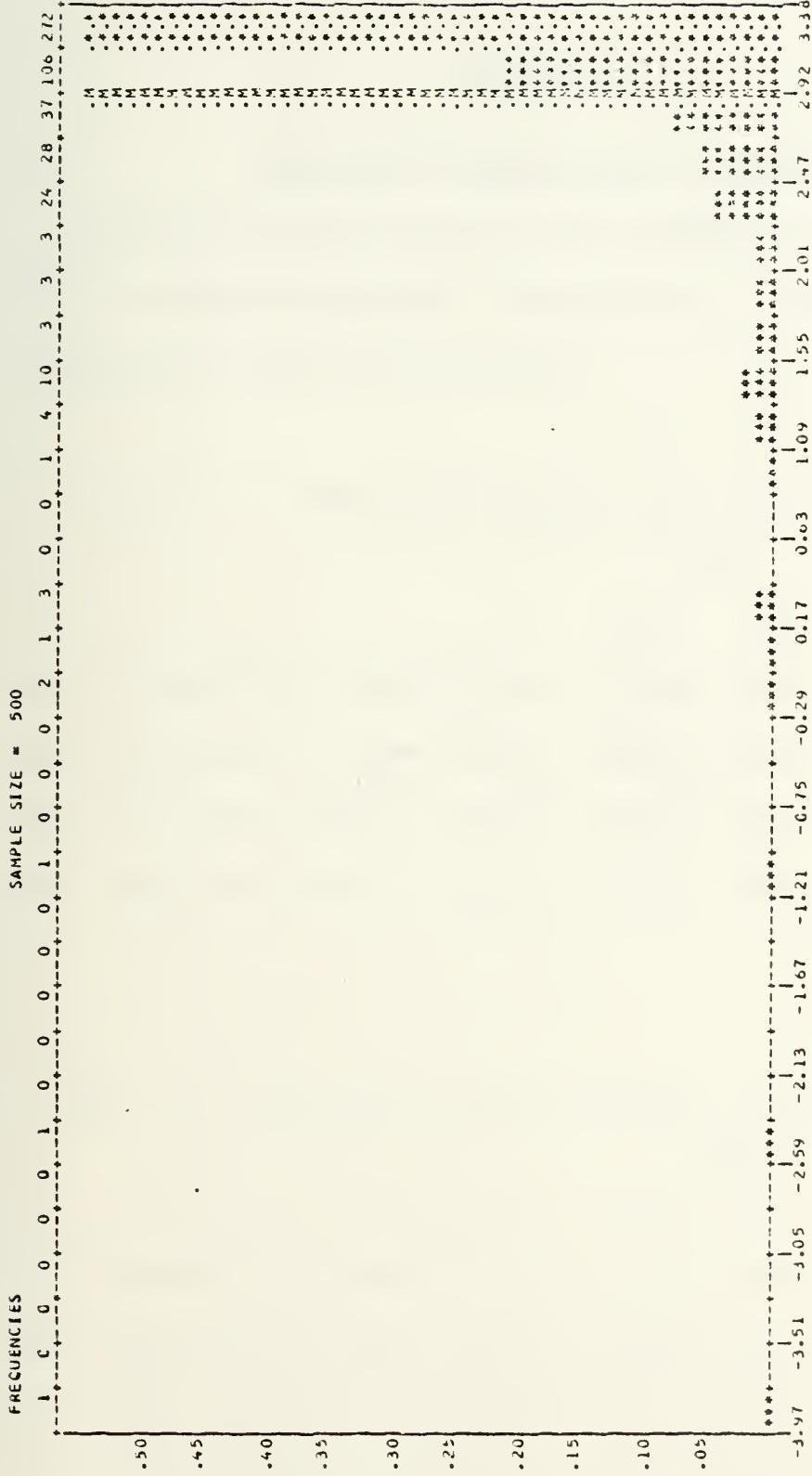


| CENTRAL TENDENCY | SPREAD | | | HIGHER CENTRAL MOMENTS | | | DISTRIBUTION | | |
|------------------|---------------|--------------|--------------|------------------------|--------------|--------------|--------------|-------|---------|
| | MEDIAN | STD DEV | COEF VAR | VARIANCE | SKNESS | KURTOSIS | BETA1 | BETA2 | MINIMUM |
| MEAN | 8.505710DE-01 | 7.65752E-01 | 7.796738E-01 | 8.668774E-01 | 43 | 3.440938E-01 | -2.03579E+00 | | |
| MEDIAN | 7.65752E-01 | 7.796738E-01 | 7.95294E-01 | 9.310625E-01 | 2.631816E+00 | 2.26320E+00 | -2.91514E-01 | | |
| TRIMMED MEAN | | | | 1.094629E-01 | 4.26320E+00 | 2.22776E-01 | -2.12027E-01 | | |
| MIDMEAN | | | | 7.316915E-01 | 5.022342E-01 | 7.69582E-01 | 7.36717E-01 | | |
| MIDRANGE | 1.231330E+00 | | | 6.541920E+00 | 3.220342E-01 | 2.619842E+00 | 2.36231E+00 | | |
| | | | | 1.155153E+00 | | | 4.503340E+00 | | |
| | | | | MIDSPREAD | | | | | |

20. Log-transform of long-tailed log-normal down times after grouping.



21. Pseudovalues of JK procedure for Exponential up and long-tailed log-normal down times after grouping.



| CENTRAL TENDENCY | SPREAD | HIGHER CENTRAL MOMENTS | DISTRIBUTION |
|------------------|---------------|------------------------------|----------------------|
| MEAN | 4.8225097E-01 | M ₃ -1.556422E-00 | MINIMUM |
| MEDIAN | 3.1975195E-00 | M ₄ 7.971553E+00 | MINIMUM QUANTILE |
| TRIMMED MEAN | 3.1653145E+00 | SKEWNESS -4.643745E+00 | 25 QUANTILE (HINGE) |
| MEADIAN | 3.1729545E-00 | KURTOSIS 3.123976E+00 | 50 QUANTILE (MEDIAN) |
| MIN RANGE | -2.928467E-01 | BETA1 -3.547096E+00 | 75 QUANTILE (HINGE) |
| MIDSPAN | 3.963064E-01 | BETA2 7.910567E+00 | 90 QUANTILE |
| | | | MAXIMUM |

22. Pseudovalue of LNLJ procedure for Exponential up and long-tailed log-normal down times after grouping.

APPENDIX D

ONE-SIDED AVAILABILITY ESTIMATION

Best and worst cases for LNLJ procedure were simulated for one-sided availability estimation, and results are shown in Tables 17 and 18 as follows;

Table 17: Exponential Up and Log-normal Down Times

| | N=15 | | N=25 | |
|-------------------------------|-------|-------|-------|-------|
| | JK | LN | JK | LN |
| Low. Availability Bound Cover | 0.999 | 1.000 | 0.999 | 1.000 |
| Upp. Availability Bound Cover | 0.933 | 0.899 | 0.966 | 0.965 |
| Avg. Lower Availability Bound | 0.889 | 0.879 | 0.910 | 0.908 |
| Avg. Upper Availability Bound | 0.966 | 0.964 | 0.966 | 0.966 |

Table 18: Exponential Up and Weibull ($k=1/2$) Down Times

| | N=15 | | N=25 | |
|-------------------------------|-------|-------|-------|-------|
| | JK | LN | JK | LN |
| Low. Availability Bound Cover | 0.900 | 0.990 | 0.914 | 0.997 |
| Upp. Availability Bound Cover | 0.966 | 0.853 | 0.955 | 0.705 |
| Avg. Lower Availability Bound | 0.844 | 0.460 | 0.381 | 0.500 |
| Avg. Upper Availability Bound | 0.972 | 0.970 | 0.975 | 0.955 |

Notice the low Average Lower Availability Bound provided by the LNLJ procedure for the Weibull ($k=1/2$). Results show that the one-sided availability estimation behaves as the two-sided availability estimation.

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